We start with energy and “work,” via calculations using Newton’s Laws and kinematics.

For motivation, we worked through a few simple setups. Generally we asked the question: if some force acts on the system over a certain distance, what speed does it have in the end? For the purpose of motivating the forthcoming laws, we calculated as follows: use Newton’s Laws to find acceleration, then use kinematics to find the speed. Since we consider forces acting over distance, we use the ‘third’ kinematics equation, which is tailored for the job: $v_f^2 = v_0^2 + 2a_x \Delta x$. Note that other than motivating the following statement (on the relation between energy and work), these examples also demonstrate its consistency and uses.

First, drop an object from a height $(H)$. After working out the acceleration, and using kinematics, we get $v_f = \sqrt{2gH}$, of course. Then we push our object instead, with a force $F$ over a distance $L$, and work out its speed (and keep it squared). Now the mass doesn’t cancel: $v^2 = 2(F/m)L$.

Since this ‘force×distance’ seems interesting (possibly a fundamental quantity!), we clean up our result a little, to have that term without any factors around; we get: $(1/2)mv^2 = FL$. With this ‘$FL$’ in mind, we go back to our free fall, and leave “$F_{gr}$” so that the mass doesn’t cancel in that calculation.\footnote{Friction: $ma = mg = ma \Rightarrow a = g$ is nice, but misleading: other forces can’t do that, mass doesn’t generally cancel.} Now we get: $(1/2)mv^2 = F_{gr}H$. We also realize that it is just as easy to allow some initial speed (this only adds the term $v_0^2$, without making anything more difficult); it also turns out that the direction of this initial speed doesn’t matter for the equations we are getting! The speed is always squared, so whether the object has been thrown up or down, it is simply $v_0^2$.

Throw: \( \frac{1}{2}mv_0^2 + F_{gr}H = \frac{1}{2}mv^2 \).

The quantity $(1/2)mv^2$ gets added to by the $FL$, force ‘working’ on the object over a distance. At this point we can drop the pretense (motivation): $(1/2)mv^2$ is kinetic energy, one of the most fundamental quantities in physics. Note that it only involves the speed and mass; it characterizes motion. Also note that speed ‘matters more,’ so to say: it is squared, while mass isn’t. The other player in the above equations is work: force acting over distance (working) brings changes in the energy of the system. But how exactly do forces ‘work’ on the system? How about pushing/pulling under angles? What with friction, acting opposite the motion? Next we worked out two more examples. We construct the same kinetic energy of an object (starting out with $v_0$) after a force has been pushing over a distance $(L)$, like before, with the force acting under an angle $(\theta)$. Then we do the same when only friction acts on our object. The results are intriguing:

- $F$ acting under angle: $\frac{1}{2}mv_0^2 + FL \cos \theta = \frac{1}{2}mv^2$.
- Friction: $\frac{1}{2}mv_0^2 − F_{fr}L = \frac{1}{2}mv^2$.

These all seem to involve ‘$FL$’ in one way or another; and there is a general way to write that, using scalar product: $\vec{F} \cdot \vec{L} = FL \cos(\vec{L}, \vec{F})$ (cosine of the angle between these vectors, traversed counter-clock-wise). This takes care of various directions under which the force may be acting. What if it changes throughout? To estimate the contribution of such force over the whole path of the system, we integrate.\footnote{In the above examples: the force is constant; first it acts under $\theta$; for friction under $180^\circ$ – opposite to motion.} Then we can state one of the deepest and biggest laws of physics.

\[ \frac{1}{2}mv_0^2 + W = \frac{1}{2}mv^2 \quad \text{where} \quad W = \int_{\text{path}} \vec{F} \cdot d\vec{l} \quad \text{(may be negative!)} \]

The initial kinetic energy of the system, and the work done on the system by external forces, result in the final kinetic energy. The forces build up the kinetic energy by doing (positive) work on the system, or may dissipate it (to heat), like friction does, by working ‘against’ the motion.

Next we will see that for many forces there are potentials, and so their work need not be explicitly calculated, since it is accounted for by (formulas for) their potential energy.
\[ \Sigma F_y = F_{gr} = m a_y \implies a_y = g \]

\[ n^2 = v_0^2 + 2 a dx \]

\[ n^2 = 2 g H \]

\[ \frac{1}{2} n^2 = g H \]

\[ \frac{1}{2} v^2 = \frac{1}{2} v_0^2 + g H \]

\[ L \]

\[ v_0 = 0 \implies n^2 = 2 \cdot \frac{F_o}{w} L \]

\[ n_0 \neq 0 \implies n^2 = n_0^2 + 2 \cdot \frac{F_o}{w} L \]

\[ \Sigma F_y = m a_y \]

\[ F_{gr} = m a_y \]

\[ \Sigma F_x = F_o \cos \theta = m ax \]

\[ a_x = \frac{F_o}{w} \cos \theta \implies \]

\[ v^2 = v_0^2 + 2 a dx \]

\[ \frac{1}{2} w v^2 = \frac{1}{2} w v_0^2 + F_o \cos \theta \cdot L \]

\[ v^2 = v_0^2 + 2 \frac{F_o}{w} \cos \theta \cdot L \]

\[ \sqrt{\frac{w}{2}} \]
In all previous examples, we had:

\[
\frac{1}{2} m v^2 \left( = \right) \frac{1}{2} m v_0^2 + \text{force} \cdot \text{worked} \over \text{over distance } v \quad (\text{something like } F \cdot L \text{ every time})
\]

\[
\frac{1}{2} m v^2 : \text{kinetic energy}
\]

\[
\frac{1}{2} m v_0^2 + \text{"work"} = \frac{1}{2} m v^2
\]

\[
\rightarrow \quad v_0 \rightarrow \quad \frac{F \cdot r}{m} \rightarrow x
\]

\[
(x) \quad F_{fr} = m a_x \Rightarrow a_x = -\frac{F_{fr}}{m}
\]

\[
N^2 = N_0^2 + 2 a x \cdot x \Rightarrow N^2 = N_0^2 - 2 \frac{F_{fr}}{m} \cdot L
\]

\[
\Rightarrow \frac{1}{2} m v^2 = \frac{1}{2} m v_0^2 - \frac{F_{fr}}{m} \cdot L
\]

The interaction transfers some of the object's kinetic energy to molecules, making them move around more rapidly, etc.; this is heat! So the effect of friction was to transfer object's kinetic energy to environment.
\[
\frac{1}{2}mV_o^2 + W = W_{kinetic}
\]

\[
W = \mathbf{F}_1 \cdot \mathbf{x} = F_1 \cos \theta \cdot \mathbf{x}
\]

\[
W = \mathbf{F}_1 \cdot \mathbf{x}
\]

Can we have the above \((\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, -\mathbf{F})\) in one formula?

Yes!

"scalar product"

vectors \rightarrow \text{number (scalar)}

\[
\mathbf{F}_1 \cdot \mathbf{X} = F_1 \cdot x
\]

"scalar product"

(or "dot" product)

force \times \text{distance}

\[
W = \sum F_i \cdot d_i
\]

\[
W = \int_0^L F \cdot d \theta
\]

Work done by force \(\mathbf{F} \) along the path \(\ell\) ("initial" to "final" spot).

So, "W" is a "scalar": a number.

\(\text{Soy, 10^4 \text{ joules}}\).

The rule to compute:

\[
W = \mathbf{F} \cdot \mathbf{x} = F \cdot x \cos \theta \rightarrow \text{angle between } \mathbf{x} \text{ and } \mathbf{F}
\]

\(\theta\) goes from \(\mathbf{x}\) to \(\mathbf{F}\) counter-clockwise.

\[
\cos(\mathbf{x}, \mathbf{F}) (x\text{-to-}F)
\]
\[ W = \overrightarrow{F}_{gr} \cdot \overrightarrow{y} = \overrightarrow{F}_{gr} \cdot \overrightarrow{H} \] (c25)

\[ W = \overrightarrow{F} \cdot \overrightarrow{x} = \int \overrightarrow{F} \cdot d\overrightarrow{x} \]

\[ \overrightarrow{F} \cdot \overrightarrow{x} = F \sin \theta = F \ell \sin \theta = H \]

\[ W_{gr} = \overrightarrow{F}_{gr} \cdot \overrightarrow{H} = \overrightarrow{F}_{gr} \cdot (y_f - y_i) \]

\[ \text{work done by gravity depends only on the change in height!} \]

"Potential."

\[ \frac{1}{2} m v_0^2 + W = \frac{1}{2} m v^2 \]

\[ \frac{1}{2} m v_0^2 + m g \cdot H = \frac{1}{2} m v^2 \]

\[ \text{"initial energy" = "final energy"} \]

\[ \text{"potential energy in gravity"} \]

(Next class!)