Let us now review the matters of how energy transforms as the system moves around; of various energy conversions; and of how our few energy and work equations relate to each other.

\[ \frac{1}{2}mv_i^2 + W_{all} = \frac{1}{2}mv_f^2, \]

can be used as:

\[ \frac{1}{2}mv_i^2 + \sum U_i + W_{np} = \frac{1}{2}mv_f^2 + \sum U_f. \]

These equations describe energy balance in physical systems, geared toward mechanics.\(^1\) The first one is a very general statement, that a change in kinetic energy is due to the work done on the system. In Newtonian mechanics, this implies that we need to know the ‘forces’ modeling external influences, and then compute the work done by them. Many forces are “conservative:” the work done by them around a closed path, when the system ends up where it started, is zero. Imagine raising something to a height (without acceleration, by a force exactly equal to gravity, so that we can barely move it upward), and then releasing it, so that it drops to where we started. What is the total work by gravity? While our force \((F_0 = mg)\) is raising it, doing the work of \(mgH\), the work by gravity is \(-mgH\); then, as it drops to the ground, the work by gravity is \(+mgH\) (calculate these!). Total work by it: a clean zero. The same goes for a spring: work done by it while being manipulated by external forces is negative, while when released it does the exact same amount of positive work. These simple observations come with some big conclusions. The work done by such forces conserves mechanical energy. Also, we see directly that by doing work against them, we build potential energy in their fields. This allows for the convenient second equation.

As the system moves around, its energy is constantly changing between various potential ‘forms’ and kinetic energy. It may be pushed up a plane, with say 10 J of kinetic energy, and as it is ascending this is gradually converted into the potential energy in gravity. Say that it reaches a flat section, at such a height so that \(mgH = 4\) J; then 6 J of kinetic energy remains. Then it may hit a spring, and compress it so that all this 6 J of energy goes into the potential energy of the spring – which then shoots it out, returning the kinetic energy to it. It may proceed downhill, now building more kinetic energy on the account of ‘cashing in’ on its gravitational potential. By the time it gets back to where it started, it has its 10 J of kinetic energy. In other words: its overall energy is conserved, merely going from one ‘form’ to another. But, what about friction? Air resistance? Rocket motors\(^2\) pushing? These aren’t accounted for by the above description. Also, these forces don’t have potentials, so the above example of energy being converted around between ‘potential’ and kinetic cannot accommodate them. At this point, we need to clear up some semantic issues.

First and foremost: energy is always ‘conserved.’ When particles are ‘created’ in high-energy collisions in accelerators, that happens because of some other particles breaking up, of ours pumping energy into the system, etc. When a box shoved across a level floor stops, we understand that the energy we ‘gave’ it (by doing work to push it up to speed) went into heating the box, the floor, the air, producing the whooshing sound, sparks as nails are hit, etc. The energy that physical systems acquire, by work done on them, is distributed around as they move and interact. Part of it goes between kinetic and various forms of potential energy, and is maintained in system’s motion; while part goes into other forms of energy, heating things up, causing structural changes, being emitted as sound or light, etc. As the system goes around, generally it has less and less speed, can reach lower heights, can compress springs less – the energy it carries gets dissipated into the environment. Often a distinction is made between mechanical energy and ‘other’ forms.

This is seen in the second equation, to an extent. If there are no influences that dissipate energy, it is said that mechanical energy is conserved. But please note that the above equation, as it stands, very clearly expresses energy conservation in general, where work by ‘other’ forces isn’t something alien, but accounts for physical processes in which energy is distributed. It is only with

\(^{1}\)In principle, ‘work by forces’ covers nearly anything, but it is often not practical beyond mechanical systems.

\(^{2}\)Some fuel is burning, releasing energy from (say) chemical bonds, and so increasing the kinetic energy.
reference to mechanical energy that one says that some energy has ‘gone to heat’ (for example); in
general, heat is of course a normal part of energy balance and transformations.

We can summarize all this in the following way: $E_i = E_f$, with $E$ being the total energy, and
where ‘initial’ and ‘final’ are of course arbitrary, any two points on the route we care to pick. In more
detail, separating potential and kinetic energy, this can be stated as: $K_i + \sum U_i + W = K_f + \sum U_f$.
In our class, $K + U$ represents “mechanical” energy.\(^3\)

Let’s consider an example, bringing one more topic into the discussion. A bullet is fired, that
hits a block of wood and stops inside, and then this system slides further, finally hitting a spring and
stopping. All this happens on an upward incline (block is $H$ above where the bullet starts).
We take it that the friction after the collision, of the block (with the bullet inside) with the surface,
is considerable and cannot be dismissed. Let us talk through energy conversions in this process.

First, the bullet ascends, and as it reaches the block it has less kinetic energy than it started
with: $(1/2)mv_0^2 = mgH + (1/2)mv_f^2$. As it hits the block with $v_f$, it enters it and is brought to a
stop, and then the block+bullet move further up, eventually hitting the spring and compressing it
(by say $\Delta x$), and finally stopping.\(^4\) As they move up an incline, they stop at an additional height
of $h$. We could say that the energy balance is: $(1/2)mv_0^2 + W = (m + M)g(H + h) + (1/2)k(\Delta x)^2$,
for the whole process, where “$W$” includes work by friction, and also represents work by forces
during the collision of the bullet with the block.

This is always possible to write, as the ‘work by forces’ governs in principle any energy transfor-
mations. But sometimes it is just (way) too hard to use; how does one model various, and extremely
rapidly varying, forces acting as the bullet is penetrating the block, and being brought to a stop
in it? Often we have to conclude that we cannot estimate how much energy went to phenomena
‘other’ than mechanical motion.\(^5\) This is when momentum conservation comes to rescue. We can
break this process up, into segments on which we can realistically use our energy balance equations,
and those events for which momentum conservation can help.

Up to the point of hitting the block the bullet undergoes: $(1/2)mv_0^2 = mgH + (1/2)mv_f^2$. The event of the (perfectly inelastic) collision is covered by momentum conservation: $mv_f = (m + M)V$.
Then the bullet+block system $(m + M)$ moves on with speed $V$, hits and compresses the spring,
stopping at the extra height $h$, so: $(1/2)(m + M)V^2 + W_f = (m + M)gh + (1/2)k(\Delta x)^2$. Note that
work by friction is included. This describes the whole process. Summarized:

$$\frac{1}{2}mv_0^2 + mgH + \frac{1}{2}mv_f^2 \rightarrow mv_f = (m + M)V \rightarrow \frac{1}{2}(m + M)V^2 + W_f = (m + M)gh + \frac{1}{2}k(\Delta x)^2.$$ 

Also, just so, let us make a problem out of this. We can measure the heights $H$ and $h$, and know
the inclination angle $\theta$; we know the coefficient of friction ($\mu_k$). We surely can measure masses $m$
and $M$. We calibrated our spring and know its $k$, and can measure how much it compresses ($\Delta x$).
Air resistance is negligible, compared to everything else. What was the speed of the bullet? What
fraction of energy went to heat and structural changes of the objects involved? Obtain formulas.\(^6\)

[In terms of “mechanical energy conservation,” all initial energy should end up converted into the final potential
energy, of the spring and gravity. But it doesn’t, part of it having gone to those ‘other’ forms of energy. The
difference between the two is how much energy went out of the system’s motion, into heat, structural changes,
etc. Dividing this by the initial total energy gives you the “fraction.”]

Note that we can account for the energy converted to heat due to friction, while we cannot
directly estimate the ‘losses’ (of mechanical energy) due to the inelastic collision.

\(^3\)Not generally: many phenomena that carry potential and kinetic energy cannot possibly be called ‘mechanical.’
\(^4\)For a moment, then the spring would shoot them out, unless it is somehow locked in that compressed position.
\(^5\)The work of friction, or of a ‘wind’ with a known force, or of a ‘thrust’ of engines (with a known force), is fine
and easy. Computing work done as a bullet negotiates a piece of wood, is beyond most models and techniques.
\(^6\)Also: go ahead and pick some reasonable numbers, and then get numbers too. Do they seem reasonable?
Questions answered in the beginning:

1. A bullet hits a block of wood, and is stopped inside. This then moves, and finally is stopped by a spring.

\[ m \cdot v_0 = (m + M) \cdot v \]

Need momentum conservation:

Momentum is conserved!

Then they move off, compress the spring and stop. For that we can use energy conservation:

\[ \frac{1}{2} (m + M) V^2 = \frac{1}{2} k \Delta x^2 \]

So we have:

\[ m \cdot v_0 = (m + M) \cdot v \]

\[ \frac{1}{2} (m + M) V^2 = \frac{1}{2} k \Delta x^2 \]

Solve for what the problem asks for.
(2) A cannon fires and recoils, to be stopped by a spring. The cannon-ball flies out...

"explosion": again, we cannot use our energy balance eqn. 2, since the explosion is much too complicated to model via forces: use momentum conservation.

\[ 0 = -M\vec{V}_1 + m\vec{v} \]

\[ \Rightarrow M\vec{V}_1 = m\vec{v} \]

Now they move off, and we can use energy conservation: the cannon is stopped by the spring.

So:

\[ M\vec{V}_1 = m\vec{v}, \quad \text{and} \quad \frac{1}{2} MV^2 = \frac{1}{2} kAx^2. \]

Solve for what is needed.

In both examples, work by friction (if needed) is calculated as usual. The only thing that is different from our 'normal' energy equations is the event for which we need momentum conservation. Before, and after, that we do normal energy-work calculations.
Example: spring pushes an object uphill, where it hits and sticks to another. How far do they land? There's friction, wind \( F_w \) is horizontal in the back, and the object is propelled by a thrust \( F_t \).

1) until it hits:

\[
\frac{1}{2} k \Delta x^2 + W_{F_0} + W_{F_1} + W_{fr} = m g H + \frac{1}{2} m_1 v_1^2
\]

\[ U_i + W = U_f + K_f \]

\[ W_{F_0} = F_0 L \cos \theta \]
\[ \sin \theta = \frac{H}{L} \implies L = \frac{H}{\sin \theta} \]

\[ W_{fr} = -F_{fr} \cdot L \]. For "\( F_{fr} \)" use Newton's Law:

\[ W_{F_1} = F_1 \cdot L \]

\[ \implies \text{Solve for } v_1 \]

2) The collision is inelastic; must only use momentum conservation:

\[ m_1 v_1 = (m_1 + m_2) v_f \]

3) They fly off with \( v_f \); projectile
Ki + Ui + W = Ke + Uf

\[ mgL + \frac{W_{Fo}}{L} + \frac{W_{fr}}{2} = mg(h + h_f) + \frac{1}{2}k(\Delta x)^2 \]

Total height (final) above ground.

\[ W_{Fo} = -F_o \cos \theta \cdot L \quad \text{and} \quad W_{fr} = -F_{fr} \cdot L \]

Use Newton's Laws

Need to find \( h_f \) via \( \Delta x \):

\[ \sin \theta = \frac{h_f}{\Delta x} \]

\[ h_f = \Delta x \sin \theta \]

\[ mgL + \frac{W_{Fo}}{L} + \frac{W_{fr}}{2} = mg(h + \Delta x \sin \theta) + \frac{1}{2}k(\Delta x)^2 \]

Solve for \( \Delta x \) (quadratic equation!).

\[ W_{Fo} = F_o \cdot L = F_o \sin \theta L \]

\[ W_{fr} = -F_{fr} \cdot L \]

\[ F_{fr} = \mu_k F_N = \mu_k F_o \cos \theta \]

Use Newton's Laws

\[ U_i + K_i + W = U_f + K_f \]

\[ O + W_{Fo} + W_{fr} = mgL + \frac{1}{2}mV_f^2 \]

Solve for \( V_f \).

Initial potential energy; we call initial level 0.
There was a question: "work by tension," etc

An object is accelerated by given \( \mathbf{A} \) over a distance (height) \( H \). What is work done by tension?

\[
W_T = \mathbf{F}_T \cdot \mathbf{H} = \mathbf{F}_T \cdot H, \quad \text{clearly.}
\]

But, what is \( \mathbf{F}_T \)? Use Newton's Laws!

\[
\mathbf{F}_T - \mathbf{mg} = \mathbf{ma}, \quad \mathbf{F}_T = \mathbf{mg} + \mathbf{ma} \Rightarrow W_T = \mathbf{mg}(g+a) \cdot H
\]

Use both indicated coordinate systems. The point: energy conservation/balance laws work with changes in energy, and this is particularly striking with \( U_{\text{grav}} \).

In system (1):

\[
w \mathbf{g} H + \frac{1}{2} m v_0^2 = w \mathbf{g} (-\Delta x) + \frac{1}{2} k \Delta x^2
\]

In system (2):

\[
w \mathbf{g} (H+\Delta x) + \frac{1}{2} m v_0^2 = \frac{1}{2} k \Delta x^2
\]

We get the same equation, as we must.

The change in gravitational potential energy is \( \Delta U_{\text{grav}} = w \mathbf{g} (H+\Delta x) \rightarrow \text{it doesn't matter how it's written.} \)