Kinematics equations: \[ x = x_0 + v_0 t + \frac{1}{2}at^2, \quad v = v_0 + at. \]

They describe a motion with constant acceleration.

Braking example, \( a = -1 \text{ m/s} \). Initial: \( x_0 = 10 \text{ m}, \ v_0 = 10 \text{ m/s} \).

\[ x(t=1s) = 10 \text{ m} + 10 \text{ m} (1 \text{ s}) + \frac{1}{2}(-1 \text{ m/s}^2) (1 \text{ s})^2 = 19.5 \text{ m} \]

\[
\begin{align*}
    t = 2 \text{ s} &: \quad x = 10 \text{ m} + 10 \text{ m} - 2 \text{ m} = 28 \text{ m} \\
    t = 5 \text{ s} &: \quad x = 10 \text{ m} + 50 \text{ m} - 12.5 \text{ m} = 47.5 \text{ m} \\
    t = 10 \text{ s} &: \quad x = 10 \text{ m} + 100 \text{ m} - 50 \text{ m} = 60 \text{ m} \\
    t = 15 \text{ s} &: \quad x = 10 \text{ m} + 150 \text{ m} - 112.5 \text{ m} = 47.5 \text{ m} \quad \text{(huh??)} \\
    t = 20 \text{ s} &: \quad x = 10 \text{ m} + 200 \text{ m} - 200 \text{ m} = 10 \text{ m} \\
\end{align*}
\]

A problem ... the distance should not start decreasing!

Why is this — math vs. physics. A very important point.
For this, we can graph the function / data

Braking, with reaction.
The car will stop, and our equation doesn’t make sense anymore. *Always* need to keep a sense of our physics, when using math.

(Try the above with different numbers – initial speed, acceleration … )

So … how far does the car get? How long until it stops?

Slowly, this gets us into solving problems.

Not just an equation to simply put numbers in.

Our equations – tools to work with:

\[ x = x_0 + v_0 t + \frac{1}{2} a t^2, \quad v = v_0 + a t \]
Reference: \( x = x_0 + v_0 t + \frac{1}{2} a t^2, \quad v = v_0 + a t. \)

The key ‘observation’: the car stops . . . and so \( v = 0. \)

The car is at this particular spot at a specific moment in time.

Let me call them . . . \( x_s \) and \( t_s \) (for stop). Then

\[
\nu(\text{at stop}) = 0 \quad \Rightarrow \quad 0 = \nu_0 + a t_s,
\]

and we can directly solve for it from the above equation!

*(Important: “\( t_s \)” is a number, a specific value for time.)*

\[
0 = \nu_0 + a t_s \quad \Rightarrow \quad t_s = -\frac{\nu_0}{a}
\]

Note, the acceleration itself was negative.
So we got the time it took to stop,

\[ t_s = -\frac{v_0}{a} = -\frac{10 \, \text{m/s}}{(-1 \, \text{m/s}^2)} = 10 \, \text{s} \]

With time to stop, we can go back to the other equation, \( x(t) \). Enter this value for time, and get the position at that time – position when stopped thus. This is the distance we wanted. (Given our coordinate system.)

At time \( t = 10 \, \text{s} \), the position is, from \( x = x_0 + v_0 t + \frac{1}{2}at^2 \) :

\[
x(t=10\,\text{s}) = 10 \, \text{m} + (10 \, \text{m/s})(10 \, \text{s}) + \frac{1}{2}(-1 \, \text{m/s}^2)(10 \, \text{s})^2\\x_s = 60 \, \text{m}
\]

We got our distance – and we got the time in the process.
How about some other values? For a different $a$? Or, other $v_0$? Would have to recalculate, go back to the beginning. Or ... not?

What we got was a *formula* for time:

$$t_s = - \frac{v_0}{a}$$

We then put this time into the $x(t)$ equation – but as a number.

Why? We don’t have to ‘ruin’ a perfectly good formula! Keep the symbol: with the above expression for $t_s$

$$x_s = x_0 + v_0 t_s + \frac{1}{2} a t_s^2 \quad \Rightarrow \quad x_s = x_0 + v_0 \left( -\frac{v_0}{a} \right) + \frac{1}{2} a \left( -\frac{v_0}{a} \right)^2$$

Carry out the algebra ... $x_s = x_0 - \frac{1}{2} \frac{v_0^2}{a}$. *(a is negative!)*
... and better yet!

Reference: \( x = x_0 + v_0 t + \frac{1}{2}at^2, \ v = v_0 + at. \)

The method we used: find time, substitute into position equation.

Can we do something like that always? In general?
Yes! And that way we’ll get something really useful.

So, the quest: eliminate time from the equations of motion.
This is a general method for ‘solving systems of equations.’

It means: ‘solve for it’ from one equation (say, the second).
Then use this expression instead of it in the other (the first).

\[
\text{‘solve for time’} \quad v = v_0 + at \quad \Rightarrow \quad t = \frac{v - v_0}{a}, \quad \text{and}
\]

\[
x = x_0 + v_0 \left( \frac{v - v_0}{a} \right) + \frac{1}{2}a \left( \frac{v - v_0}{a} \right)^2 \quad \text{note: time’s gone!}
\]
Finally

Clearing up that algebra (*excellent* exercise), we get

\[ x = x_0 - \frac{1}{2a}(v_0^2 - v^2), \quad \text{or:} \quad 2a(x - x_0) = v^2 - v_0^2 \]

It is convenient to write this as

\[ v^2 = v_0^2 + 2a(x - x_0) \]

\((x - x_0)\) is simply displacement. It is often labeled “\(\Delta x\)”. With this, \((x - x_0) = \Delta x\), we have equations of kinematics:

\[ x = x_0 + v_0 t + \frac{1}{2}at^2 \]
\[ v = v_0 + at \]
\[ v^2 = v_0^2 + 2a\Delta x \quad (\Delta x = x - x_0) \]

Remember, the third equation is a combination of the first two. We got it directly from them, by eliminating time.
Comments

- All equations we’ll need for weeks! The rest ... ? Problems.

- Relevant material in the book: Ch 2 section 4, and up to it. Fill-in on the reading. There is a lot, but most of it is so basic that one just must have read that. So do it now, skip if stuck.

- A car goes 60 mi/hr when it starts braking at the rate of 5 m/s². Stopping distance? What is it with harsher braking? Say, twice harsher? Five times?

- Remember the trouble with shrinking distances in our first example for ‘braking with reaction’ ... ? Can you come up with an example (a physical system – a ‘situation’) that will be described correctly and fully by the equation we studied! (So that there is no need to say ‘only up to that point’ . . . )