Vectors and components

For a vector $\vec{V}$, its components in a given Cartesian coordinate system are:

$$V_x = V \cos \theta, \quad V_y = V \sin \theta$$  \hspace{1cm} \text{(comp)}

Label $V$ stands for the magnitude: $V \equiv |\vec{V}|$. The magnitude and angle relate to the components as:

$$|\vec{V}| = \sqrt{V_x^2 + V_y^2}, \quad \tan \theta = \frac{V_y}{V_x}$$  \hspace{1cm} \text{(vec)}

Angle calculated as above may need adjustments, regarding its sign and factors of $\pm \pi$.

Kinematics

Equations of kinematics, integrated from a constant acceleration.

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$  \hspace{1cm} \text{(k1)}

$$v_x = v_{0x} + a_x t$$  \hspace{1cm} \text{(k2)}

$$v_x^2 = v_{0x}^2 + 2 a_x \Delta x, \quad \text{where} \quad \Delta x = (x - x_0)$$  \hspace{1cm} \text{(k3)}

The analogous set holds involving the $y$-components of position, velocity, and acceleration.

Newton’s Laws

The vector equation for Newton’s first and second Law: $\sum \vec{F}_i = m\vec{a}$ (sum over all forces).

Written out in components:

$$\sum_i F_{ix} = m a_x$$  \hspace{1cm} \text{(Sums are over all components.)}$$

$$\sum_i F_{iy} = m a_y$$  \hspace{1cm} \text{(NL)}

Friction. $F_N$ is the “normal force.” Indices “s” and “k” stand for “static” and “kinetic.”

$$F_{fr}^{(s)} \leq \mu_s F_N \quad \text{ (static)}$$
$$F_{fr}^{(k)} = \mu_k F_N \quad \text{ (kinetic)}$$  \hspace{1cm} \text{(fric)}

Direction is opposite to motion (kinetic), or opposite to where it would go without friction (static).

Gravity. Gravitational force, on an object of mass $m$, close to the surface of Earth.

$$F_{grav} = mg \quad \text{ (g = 9.8 m/s}^2)$$  \hspace{1cm} \text{(grav)}

Direction: Earth’s gravitational force on an object of mass $m$ is toward Earth’s center.

Quadratic equation. $ax^2 + bx + c = 0$, where $a, b, c$ are real coefficients, has solutions:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$  \hspace{1cm} \text{(qdr)}

The solutions can be: both real (possibly equal), if $b^2 - 4ac \geq 0$; or complex otherwise.