Disentangling Preferences and Expectations in Stated Preference Analysis with Respondent Uncertainty: The Case of Invasive Species Prevention

Bill Provencher, corresponding author
Professor
Department of Agricultural and Applied Economics
University of Wisconsin, Madison
427 Lorch St.
Madison, WI 53706
(608) 262-9494
rwproven@wisc.edu

David J. Lewis
Assistant Professor
Department of Economics
University of Puget Sound
1500 N. Warner St.
Tacoma, WA 98416
(253) 879-3553
djlewis@pugetsound.edu

Kathryn Anderson
Graduate Student
Department of Sociology
University of Wisconsin, Madison
1710 University Ave., Room 287
Madison, WI 53706
(608) 890-0337
kganderson@wisc.edu

Running Title: Preferences and Expectations in Stated Preference Analysis

9/26/2011
Acknowledgements: We received outstanding advice from the manuscript’s referees. We thank Eric Horsch, Steven Chambers, and Kate Zipp for research assistance. We thank seminar participants at the AERE sessions in the 2009 AAEA annual meeting for helpful comments. Funding was provided by the National Science Foundation via the Long Term Ecological Research Network—North Temperate Lakes Site and the program in Coupled Human and Natural Systems, and by the University of Wisconsin Sea Grant.
Abstract

Contingent valuation typically involves presenting the respondent with a choice to pay for a program intended to improve future outcomes, such as a program to place parcels into conservation easement, or a program to manage an invasive species. Deducing from these data the value of the good (or bad) at the core of the program—the welfare gain generated by a parcel of conserved land, for instance, or the loss incurred by a species invasion—often is not possible because respondent preferences are conflated with their expectations about future environmental outcomes in the absence of the program. This paper formally demonstrates this conundrum in the context of a standard contingent valuation survey, and examines the use of additional survey data to resolve it. The application is to the prevention of lake invasions by Eurasian Watermilfoil (*Myriophyllum spicatum*), an invasive aquatic plant that is present in many lakes in the northern U.S. and Canada and a possible threat to many more. Respondents are shoreline property owners on lakes without Eurasian Watermilfoil. The estimated per-property welfare loss of a lake invasion is $30,550 for one model and $23,614 for another, both of which are in reasonable agreement with estimates obtained from a recent hedonic analysis of Eurasian Watermilfoil invasions in the study area (Horsch and Lewis 2009), and from a companion contingent valuation survey of shoreline property owners on already-invaded lakes.

**Key Words:** Stated preferences, expectations, respondent uncertainty, contingent valuation, species invasions
I. Introduction

Well-designed stated preference surveys often present respondents with a choice to pay for a program to avoid unfavorable events, or to induce favorable events. Asking respondents about their willingness to pay for a program to affect an event, rather than directly asking their willingness to pay for the outcome, is preferred as a matter of presenting a realistic valuation scenario, thereby minimizing so-called “hypothetical bias”. So, for instance, rather than asking respondents about their willingness to pay to prevent the extinction of a species, respondents are asked about their willingness to pay for a particular program intended to assure the species is preserved. Instances of such program-focused valuation studies abound in the contingent valuation literature, and cannot be enumerated here, though a sampling pertaining to environmental valuation over the years ranges from the reduction of old growth timber harvests to preserve the northern spotted owl (Rubin et al. 1991), to a phosphorus reduction program to improve water clarity (Stumborg et al. 2001), to the preservation of urban open space (Kovacs and Larson 2008).

Deducing from these data the value of the event that is the focus of the program can be problematic because expressions of willingness to pay for the program are conflated with expectations about future outcomes in the absence of the program. With reference to our species preservation example, respondents might place a low value on a preservation program not because they place a low value on the species, but rather because they don’t believe the species will become extinct in the absence of the program. The analyst can account for this by explicitly stating in the survey the outcome in the absence of the program (“Without the program, species X will become extinct in the next 5 years”), but this likely is not sufficient when the respondent has alternative beliefs about—or alternative conflicting information on—the counterfactual
outcome for the environmental good in question. Perhaps the most salient example is environmental damage due to climate change. In a 2010 posting on its website, The Nature Conservancy reports that only 18% of Americans “strongly believe that climate change is real, human-caused and harmful”.

1 Clearly a contingent valuation survey concerning a program to mitigate a particular environmental damage due to climate change cannot rely on assertions in the survey of the damage to unfold in the absence of the program.

In many instances the analyst may find it perfectly acceptable to estimate only the net benefit of the program developed in the contingent valuation scenario, but often the “core good” that is the focus of the program is the true subject of the valuation analysis, and extracting the value of this good from the valuation of the program—obtaining a “portable” value, in other words—is important in benefit transfer and understanding the welfare implications of changes in environmental policies and programs. If the analyst is truly interested in the welfare gain associated with an environmental good, or the welfare loss associated with an environmental bad, then it is critical to understand the respondent’s judgment about the outcomes in the absence of the program. Although we suspect that this issue is well understood by analysts engaged in nonmarket valuation, we know of no attempts in the environmental economics literature to explicitly disentangle expectations and preferences. In a review of the contingent valuation method, Boyle (2003) observes, “Physical descriptions of changes in resource conditions frequently are not available. In this case, contingent valuation questions often are framed to value the policy change. With vague or nonexistent information on the resource change, survey respondents are left to their own assumptions regarding what the policy change will accomplish”

1 http://www.nature.org/initiatives/climatechange/features/art26253.html. Available as of July 31, 2010. Results are based on The American Climate Values Survey, an ecoAmerica project conducted by SRIC-BI, and sponsored by the Alliance for Climate Protection, the Department of Conservation of the State of California, NRDC, and The Nature Conservancy.
He concludes, “This issue really has not been directly and extensively addressed in the contingent valuation literature and deserves more consideration” (p. 118).

Disentangling expectations and preferences is the primary motivation for the large and growing literature on the measurement of subjective probability distributions. Good recent surveys of the literature are presented in Manski (2004), Hurd (2009), and Delavande, Gine, and McKenzie (2011). Hurd (2009) observes,

“The objective of a theoretical or empirical investigation into intertemporal choice is to estimate preferences…However, such an investigation requires information about the probability distribution that the decision-maker used in coming to his or her choices because choice outcomes can be explained either by preferences or by subjective probability distributions (Manski 2004).” (pg. 544).

The primary contribution of this paper is to formally examine the conflation of expectations and preferences in contingent valuation experiments, and to argue that without additional information, separately identifying expectations and preferences requires strong assumptions and is unlikely to generate estimates that are robust to the specification of the willingness to pay (WTP) function. We show that additional survey data can resolve the identification problem, and for a particular class of events that are common in contingent valuation studies—binary events such as the extinction of a species, the preservation of a parcel of land, the invasion of a lake by a non-native species, and so forth—this resolution is especially clean and cheap (in terms of the data requirement), though it does require parametric assumptions about how the respondent’s subjective probability of an event evolves over time. The empirical application is the estimation of willingness to pay to prevent the invasion of freshwater lakes by Eurasian Watermilfoil (*Myriophyllum spicatum*; hereafter simply “Milfoil”),

---

2 Broadly understood, a “physical description” of an environmental change includes the time profile of the change. So, for instance, whereas the presence/absence of an aquatic species invasion is well-defined and easily described, the *timing* of an invasion typically is not.
an exotic aquatic weed that is present in many northern lakes in the U.S. and Canada and a possible threat to many more. Milfoil invasions have become a major environmental nuisance in freshwater lakes, having been characterized as “among the most troublesome submersed aquatic plants in North America” (Smith and Barko 1990, p. 55). The valuation exercise is applied to households with shoreline property on lakes without Milfoil, and the elicitation format is a referendum on a program to prevent a Milfoil invasion.

The most closely related papers in the valuation literature concern the effect on stated preferences of the information presented to respondents. Kataria et al. (2012) examine stated choice behavior when respondents do not agree with the information provided in the survey. The authors state, “Our study belongs to a small but increasing bulk of the [choice experiment] literature with the common objective to assess how subjective beliefs of the proposed scenario in the survey effects the retrieved welfare estimates”. Other studies in this literature include Adamowicz et al (1997), Tkac (1998), Blomquist and Whitehead (1998), Hoehn and Randall (2002), De Shazo and Cameron (2005), and Burghart et al. (2007), among others. The analysis presented in this paper differs from this literature in that the scenario presented to respondents does not state the counterfactual outcome (that is, the outcome in the absence of the Milfoil prevention program). Rather, we present the current rate of Milfoil invasions of lakes in the study area and the consequences of an invasion, and query respondents about their belief about the probability of an invasion on their lake in the next ten years. We then present a referendum scenario for a program to prevent a Milfoil invasion on the respondent’s lake. The econometric model of the choice experiment uses as data the respondent’s subjective probability of an invasion in the absence of the program. In other words, we deal with subjective beliefs about the counterfactual by directly incorporating those beliefs in the econometric model.
An additional contribution of this paper is that respondents are asked to report the *probability* that they would vote “yes” on the referendum were it to actually occur. More precisely, we ask respondents to choose from a complete set of probability categories (0-10%, 10-20%, etc.) and develop an econometric model that explicitly incorporates respondent probabilities in the estimation of the willingness to pay function. This approach allows respondents to directly express uncertainty in regard to their stated preferences. Other analyses have incorporated respondent uncertainty in the valuation question by using follow-up questions to a dichotomous yes/no question, including a continuous 0-100% certainty scale (Li and Mattsson 1995); a 10-point scale where 10 is “very certain” (Champ et al. 1997; Champ and Bishop 2001; Samnaliev et al. 2006; Blomquist et al. 2009); and a set of polychotomous expressions of uncertainty, such as “probably sure”, “definitely sure”, etc. (Blumenschein et al. 1998; Evans et al. 2003). Another set of papers frames the *initial* valuation question to allow for uncertainty, using the polychotomous choice format (Ready et al. 1995, Welsh and Poe 1998), or a 10-point certainty scale (Berrens et al. 2002).³ Our method of accounting for respondent uncertainty is distinguished from the above analyses by directly eliciting, and econometrically incorporating, respondents’ expressed probabilities of a “yes” vote on the referendum. This approach avoids the strong assumptions necessary to map imprecise expressions of uncertainty into probability space, either by direct re-coding (e.g. Champ et al. 1997) or by econometric methods that impose the assumption that all respondents map the uncertainty scale into probability space in the same way (e.g. Moore et al. 2010). Our method of eliciting probabilistic responses is more closely related to the stochastic payment card approach developed by Wang and Whittington (2005). The primary difference is that our approach asks respondents to

³ Berrens et al. (2002) actually split the sample, using the scale as the *primary* valuation question for part of the sample, and using it as a follow-up to a standard dichotomous choice question for the rest of a sample.
evaluate how they would vote in a referendum if the annual cost was $X, and follows with another referendum question with a different annual cost, and explicitly accounts for the categorical presentation of probabilities, while Wang and Whittington (2005) conducted a payment card survey and require respondents to choose one of 11 probability levels (0% to 100% by steps of 10%).

In the empirical application to shoreline property owners on lakes in northern Wisconsin, we find that the annual welfare loss from an invasion is about $1800 per shoreline property (it varies across model specifications). Importantly, our results illustrate the consequence of conflating expectations and preferences, as the estimates of the annual willingness to pay for a program to prevent milfoil invasions is approximately $570, significantly less than the estimated annual welfare loss of an invasion. The difference between the estimate of the annual welfare loss of an invasion and the annual willingness to pay for a prevention program reflects the fact that most respondents do not believe that Milfoil invasion of their lake is imminent; most believe, in fact, that the probability of an invasion within the next ten years is less than 30%.

In a convergent validity exercise, we compare the estimated welfare loss above to those of two other analyses. The first is a companion contingent valuation analysis concerning Milfoil control (as opposed to Milfoil prevention that is the focus of this paper), in which shoreline property owners on already-infested lakes were asked about their willingness to pay for a program to virtually eliminate Milfoil. The second is a recent hedonic analysis of Milfoil invasions in the study area conducted by Horsch and Lewis (2009). All estimates considered, the capitalized welfare loss from a Milfoil invasion appears to be in the range of $20,000 to $35,000 per shoreline property, and we find reasonable agreement in valuation estimates across the three analyses.
II. A Model of Expectations and Preferences, and Implications for Survey Design

In this section we develop a simple model of willingness to pay (WTP) for a program to create/maintain an environmental good for which WTP depends on the direct value of the good, and the individual’s judgment about the probability of enjoying the good in the absence of the program intervention. The model applies to a binary provision problem and serves as the basis of the econometric model employed in our empirical analysis. For expositional reasons we focus on a Milfoil invasion of a lake, but the basic model applies to any binary provision problem (species preservation, conservation of a particular land parcel, etc.).

Let $L(z)$ denote the annual loss from a Milfoil invasion, where $z$ denotes a vector of variables affecting household utility, including household characteristics, and let $P(x)$ denote the probability of an invasion during the year, where $x$ is a vector of lake characteristics affecting the probability of a successful invasion, such as public access to the lake and lake chemistry.\(^4\) The willingness to pay to assure no invasion in the initial year (year 0) is equal to the expected loss from an invasion in the initial year, $P(x) \cdot L(z)$. Conditional on no invasion in year 0, the willingness to pay to assure no invasion in year 1 is equal to this same value discounted. The probability of no invasion in year 0 is simply $1 - P(x)$, and so the unconditional WTP to assure no invasion in year 1 is (where we suppress function arguments for clarity), $\frac{1-P}{1+r} \cdot P \cdot L$. By extension, the annual WTP to assure no Milfoil invasion is:

\(^4\) Similar to many aquatic invasive species, Milfoil is generally spread from lake to lake by the movement of recreational boaters (Vander Zanden et al. 2004). The species becomes attached to boats, propellers, or trailers, and can spread when boaters visit multiple lakes.
\[ WTP(x, z, r) = P \cdot L + \frac{1 - P}{1 + r} \cdot P \cdot L + \left( \frac{1 - P}{1 + r} \right)^2 \cdot P \cdot L + \left( \frac{1 - P}{1 + r} \right)^3 \cdot P \cdot L + \ldots \]

\[ = P(x) \cdot L(z) \cdot \sum_{t=0}^{\infty} \left( \frac{1 - P(x)}{1 + r} \right)^t = G(x, r) \cdot L(z) . \]  

Assuming that the discount rate is high enough that an infinite time horizon provides a good approximation of \( G(x), \) the expression in (1) simplifies to,

\[ WTP(x, z, r) = \frac{P(x)(1+r)}{r+P(x)} \cdot L(z) . \]  

The issue for the analyst is how to separately identify the loss function \( L(z) \) from data generating \( WTP(x, z, r); \) it is this loss function that is the economic basis for evaluating management policies. In principle at least, it would appear possible to identify \( L(z), \) even in the case where \( z \) and \( x \) share variables, in part because of the nonlinearity in (2), and also because of the nonlinearity implied by bounding \( P(x) \) between 0 and 1. \(^6\) But relying on such nonlinearities for identification would generate estimates of \( L(z) \) that are unlikely to be robust to alternative specifications of the functional forms of \( L(z) \) and \( P(x), \) and for some specifications of \( L(z) \) and \( P(x) \) identification is not possible. This dilemma can be resolved by asking respondents in the survey one or several questions concerning their subjective probability for the environmental event in the absence of the program. Importantly, the response need not be correct in a predictive sense; for the purpose at hand—disentangling preferences and expectations—it is enough that the stated probability of the event merely accords with the respondent’s subjective probability.

---

\(^5\) A natural question concerns our use of an infinite horizon. If the environmental good of interest is fully capitalized into land values, then our use of the infinite horizon is appropriate as landowners can always resell land. Previous work by Horsch and Lewis (2009) has shown that milfoil invasions are capitalized into shoreline land values.

\(^6\) That \( x \) and \( z \) share variables is clearly the case with Milfoil invasions because many of the variables of \( x \) that make a lake vulnerable to an invasion—the state of eutrophication and availability of public access, for instance—which also affect the respondent’s utility function, which is the basis of the loss function.
In our empirical analysis we asked respondents to state the probability of a Milfoil invasion on their lakes sometime in the next 10 years, with available responses presented as categories in 10% increments: 0-10%, 10-20%, etc, as follows:7

Based on the information just provided, and your previous understanding of milfoil invasions, what would be your best guess of the percent chance that your lake will become infested with milfoil within the next ten years?

- 0-10%
- 10-20%
- 20-30%
- 30-40%
- 40-50%
- 50-60%
- 60-70%
- 70-80%
- 80-90%
- 90-100%

Figure 1 presents a histogram of results, indicating substantial variation in respondents’ expectations of future milfoil invasions.8 Importantly, the majority of the sample does not view a milfoil invasion as imminent, as 75% of respondents think there is less than a 50% chance of their lake being invaded in the next 10 years.

Let \( p_j \) denote the midpoint of the probability category chosen by respondent \( j \). Then assuming the annual probability of an invasion is constant, \( P(x)=p_j \), we have the following relationship between the stated 10-year probability of an invasion and the annual probability:

\[
p_j = P_j + (1 - P_j) \cdot P_j + \ldots (1 - P_j)^9 P_j = 1 - (1 - P_j)^9, \tag{3}
\]

where \((1 - P_j)^9\) is the probability that an infestation does not occur in the first 10 years. Solving for \( P_j \) we find \( P_j = 1 - (1 - p_j)^{1/9} \). So, for instance, if the respondent states that there is a 50% chance of an invasion in the next 10 years, the annual chance is 7.4% (\( P=.074 \)), and if the respondent states there is a 5% chance in the next ten years, the annual chance is 0.57% (\( P=.0057 \)). Using this reported value for \( P(x_j) \) in (2) yields,

---

7 We chose a 10-year horizon because it seemed a sufficiently long horizon to elicit a positive probability of invasion, without being so long as to make the invasion a sure thing.

8 In a discrete-choice analysis of the co-variates that condition responses to this question, we find that landowners are more likely to expect an invasion if they reside on larger lakes, or on lakes with public access (regression results available upon request). Since large lakes with public access are likely to receive the most boat traffic – and hence have a higher probability of becoming invaded with milfoil – responses indicate a survey sample that is well informed of milfoil invasions.
\[ WTP_j(x, z) = \frac{p_j(1+r)}{r+p_j} \cdot L(z_j) \]  

(4)

This is the model that forms the basis for our econometric analysis.

III. Econometric Model with Respondent’s Stated Probability

In the empirical analysis, each survey respondent is presented with a program to prevent a Milfoil invasion on his or her lake, and is then presented with a referendum to apply the program at a household cost of \( t \) dollars per year. Details of the referendum scenario are presented in section IV. With respect to the econometric modeling, the important unique feature of the scenario is that, rather than the conventional approach of giving respondents a binary “Vote Yes/Vote No” choice on the referendum, we asked them about their probability of voting “yes” were the referendum to actually arise. Many prior stated preference analyses have noted the importance of respondent uncertainty in referendum questions (e.g. Li and Mattsson 1995, Welsh and Poe 1998, and Alberini et al. 2003).

3.1 Econometric Model with One Valuation Question per Respondent

To answer a referendum question, respondents presumably compare \( t_j \) to their annual willingness to pay to assure no Milfoil invasion, \( WTP_j \). Under the typical scenario in the literature, the respondent votes “yes” if \( WTP_j > t_j \), or, using the formulation in (4), if \( \frac{p_j(1+r)}{r+p_j} \cdot L(z_j) > t_j \). Recent evidence, though, indicates that often respondents are uncertain about how they would behave were the contingent scenario to actually arise; see, for instance, the recent set of papers discussing the use of an “uncertainty scale” in contingent valuation (Champ and Bishop 2001, Champ et al 2002, Moore et al 2010). In the current context, it is possible that even after accounting for the (subjective) probability of a Milfoil invasion, respondents are uncertain about their referendum choice. This likely reflects a number of factors, such as the
possibility that the program will involve higher or lower costs, serve to prevent invasions by other species, and so forth. To address this uncertainty we state the willingness to pay for the prevention program, $WTP_j$, as:

$$WTP_j^{PP} = WTP_j - \varepsilon_j = \frac{P_j(1+r)}{r+P_j} \cdot L(z_j) - \varepsilon_j. \quad (5)$$

The probability of voting “yes” on the referendum is then,

$$\pi_j = Pr\left(WTP_j > t_j\right) = Pr\left(\frac{P_j(1+r)}{r+P_j} \cdot L(z_j) - t_j > \varepsilon_j\right). \quad (6)$$

We assume that the error term is logistically distributed with scale parameter $\sigma$, in which case we have,

$$\pi_j = \frac{\exp \left(\frac{P_j(1+r)}{r+P_j} \cdot \frac{t_j}{\sigma}\right)}{1+\exp \left(\frac{P_j(1+r)}{r+P_j} \cdot \frac{t_j}{\sigma}\right)} \cdot (7)$$

Different respondents facing the same annual cost $t$ and possessing the same characteristics $z$ are likely to arrive at different values of $\pi_j$ due to unobserved differences between them. To account for this we expand $L(z_j)$ to the linear form,

$$L(z_j) = \beta z_j + \nu_j \quad , \quad (8)$$

where $\nu_j$ is an individual-specific constant known by the respondent, but unobserved by the analyst. Substituting (8) into (7) yields the form,

$$\pi_j = \frac{\exp \left(\frac{P_j(1+r) \beta z_j + \nu_j}{r+P_j} - \frac{t_j}{\sigma}\right)}{1+\exp \left(\frac{P_j(1+r) \beta z_j + \nu_j - \frac{t_j}{\sigma}}{r+P_j}\right)} \cdot (9)$$

In the survey the respondent is not asked for the value of $\pi_j$, but is instead presented with probability categories, 0-10%, 10-20%, etc. Consequently the analyst does not observe $\pi_j$, but

---

9 Alternatively, we could assert that respondent uncertainty is due solely to uncertainty about the loss function, and “feed” the error through the model development beginning with equation (1). This generates a much more complicated econometric structure, and is no better rationalized than the simple approach we take here.
does observe the probability category chosen by respondent \( j \). Defining the lower bound of the category by \( \pi_{jL} \) and the upper bound by \( \pi_{jH} \), we have from (9),

\[
\pi_{jL} = \frac{\exp\left(\frac{P_j(1+r) \beta z_j + v_j}{\sigma} - t_j \right)}{1 + \exp\left(\frac{P_j(1+r) \beta z_j + v_j}{\sigma} - t_j \right)} < \pi_{jH} \quad .
\] (10)

Note that the logit expression in (10) is monotonically increasing in the stochastic term \( v_j \). It follows that algebraically manipulating (10) (see appendix 1) identifies bounds on \( v_j \):

\[
t - \frac{P_j(1+r)}{\sigma + P_j} \beta z_j + \sigma \ln \left[ \frac{\pi_{jL}}{(1-\pi_{jL})} \right] < v_j < t - \frac{P_j(1+r)}{\sigma + P_j} \beta z_j + \sigma \ln \left[ \frac{\pi_{jH}}{(1-\pi_{jH})} \right] .
\] (11)

From the perspective of the analyst, \( v_j \) is a random variable, and so the probability that respondent \( j \) chooses the probability category \( C_j \) in responding to the referendum question is implicitly defined by the inequalities in (11). In the case where \( v_j \) is distributed logistically with scale parameter \( \varphi \) this probability can be explicitly stated (see appendix 2 for derivation),

\[
\Pr(C_j) = \frac{1}{1 + \exp\left(\frac{P_j(1+r) \beta z_j - t}{\varphi} \right) \left[ \frac{(1-\pi_{jH})}{\pi_{jH}} \right]} - \frac{1}{1 + \exp\left(\frac{P_j(1+r) \beta z_j - t}{\varphi} \right) \left[ \frac{(1-\pi_{jL})}{\pi_{jL}} \right]} .
\] (12)

The sample likelihood function is the product of these probabilities.

### 3.2 Econometric Model with Multiple Valuation Questions per Respondent

In the survey conducted for the empirical application, respondents were asked at least one follow-up referendum question with a different bid level (this is discussed in detail in the next section). These follow-up questions complicate the econometric model. From (11) the analyst can bound the unobserved portion of WTP, \( v_j \), conditional on the estimated values of \( \beta \) and \( \sigma \). Quite likely the analyst will find that in at least several cases the bounds defining \( v_j \) on the first
question do not overlap the bounds on the second question. To avoid this contradiction, we allow \( v_j \) to vary across the referendum questions, reflecting temporal variation in WTP, variation in cognition, and errors in the reduced-form approximation of complex behavior.

Specifically, letting \( k \) denote the survey question, we specify,

\[
v_{jk} = \gamma_j + \omega_{jk},
\]

where \( \omega \) is iid across respondents and questions, and \( \gamma_j \) is iid across respondents. This amendment establishes a random effects model in which survey responses are correlated via \( \gamma_j \).

Estimation of this model requires simulation of the likelihood function. In our estimation, \( \omega \) is logistically distributed with scale parameter \( \varphi \), and \( \gamma_j \) is normally distributed with standard error \( \theta \). We denote by \( C_{jk} \) the probability category chosen by respondent \( j \) on referendum question \( k \).

Conditional on \( \gamma_j \), the probability of this choice is:

\[
Pr(C_{jk} | \gamma_j) = \frac{\frac{1}{1 + \exp \left( \frac{P_j(1+r) - (\beta z_j + \gamma_j) - t_k}{\varphi} \right) \left( \frac{1 - \pi_{j,kH}}{\pi_{j,kH}} \right)^{\sigma/\varphi}} - \frac{1}{1 + \exp \left( \frac{P_j(1+r) - (\beta z_j + \gamma_j) - t_k}{\varphi} \right) \left( \frac{1 - \pi_{j,kL}}{\pi_{j,kL}} \right)^{\sigma/\varphi}}}{1 + \exp \left( \frac{P_j(1+r) - (\beta z_j + \gamma_j) - t_k}{\varphi} \right) \left( \frac{1 - \pi_{j,kH}}{\pi_{j,kH}} \right)^{\sigma/\varphi}},
\]

where \( \pi_{j,kL} \) and \( \pi_{j,kH} \) are the lower and upper bounds, respectively, of category \( C_{jk} \). The likelihood of the observed responses to the survey question requires integrating out \( \gamma_j \), which can be accomplished through simulation by taking \( L \) draws randomly from the normal distribution of \( \gamma_j \), generating an approximation of the likelihood function:

\[
P_{r Sim} \left( C_{j1} \ldots C_{jK_j} \right) = \frac{1}{L} \sum_{l=1}^{L} \prod_{k=1}^{K_j} Pr(C_{jk} | \gamma_j^l),
\]

where \( K_j \) is the number of valuation questions answered by respondent \( j \). The full-simulated log-likelihood function over \( J \) respondents is \( \sum_{j=1}^{J} \log \left[ P_{r Sim} \left( C_{j1} \ldots C_{jK_j} \right) \right] \), and estimation now includes estimating \( \theta \), the standard error of the normal distribution of the random effect \( \gamma_j \).
IV. Empirical Analysis: The Issue and the Data

Milfoil is an aquatic invasive species that has become a major nuisance in the lake country of the northern U.S. and Canada. It is spread by boaters who inadvertently carry propagules attached to their boats, anchors and trailers from lake to lake. It is blamed for “clogging” infected lakes, interfering with a lake’s ecology, and creating bad odors as the “mats” of plant matter decompose in the summer heat. Milfoil entered Wisconsin waters in the 1960s and has spread to all major water bodies (Mississippi River, St. Croix River, Wisconsin River, Lake Michigan, Lake Superior) and over 500 lakes in the state. In the survey of northern Wisconsin shoreline property owners conducted in this study, 92% of respondents whose lakes were not yet colonized by Milfoil stated they were “somewhat” (50%) or “very” (42%) familiar with the issue of Milfoil invasions.

Data for the analysis is from a web and mail survey of a sample of lakeshore property owners in Vilas County, Wisconsin, the heart of northern Wisconsin’s lake district. Sample property owners were initially surveyed in the summer of 2005, with a follow-up survey—including the Milfoil CV questions—administered in the early fall of 2008. The sample was drawn from waterfront property owners as identified from Vilas County tax rolls. The sampling of properties was not random, but instead favored properties on smaller lakes to assure adequate representation of such lakes, though we found no statistical effect of lake size on WTP.

The 2008 phase of the survey was piloted on a sample of 200 shoreline property owners during the late summer. In the full sample the survey protocol differed across sample households depending on whether the household responded to the original 2005 survey. Sample households that had responded to the 2005 survey received an initial contact letter requesting completion of the web-based survey, a follow-up postcard reminder, a follow-up hard copy of the survey
instrument, a follow-up postcard reminder, and a final follow-up hard copy of the instrument. Sample households that had not responded to the 2005 survey received the initial contact letter and three follow-up reminder letters. To reduce survey costs, hard copies of the instrument were not sent to 2005 non-respondents.

Overall, 2955 households were contacted in the 2008 survey, with 1565 (53%) providing usable responses. In the analysis the sample size concerning willingness to pay for Milfoil prevention is considerably less than the response total of 1565, for two reasons. First, not all households received a Milfoil CV question because this question was part of a larger research effort concerning the value of freshwater lake ecosystem services, and some households received different contingent valuation questions concerning a lake’s green frog population and fishing quality. Second, the nature of the Milfoil valuation question posed to a household depended on the state of the invasion on the responding household’s lake. Households on a lake that was already invaded received a Milfoil control question, whereas households on a lake not yet invaded received the Milfoil prevention question.

The scenario for the Milfoil prevention question was a lake-wide referendum for a prevention program that would make it “highly unlikely” that Milfoil would invade the lake. The scenario design followed conventional protocols for contingent valuation. Respondents were told of the consequences of a Milfoil invasion, including that in some lakes an invasion is barely noticeable while in others it is a major nuisance, and that Milfoil has been found in 20 lakes in Vilas County (about 1.5% of all lakes), and is spreading in the county at an average of 1.6 lakes per year. The scenario used the best available science on Milfoil prevention, emphasizing the use of boat-washing stations at boat ramps, paid staff to inspect boats entering and leaving the most popular lakes, and educational literature. The scenario was presented after
a short “cheap talk” script emphasized that although the scenario was hypothetical, it was
important for respondents to think about their voting behavior were the scenario to actually
unfold.

As already discussed in section 2, and unlike the usual referendum scenario where
respondents are asked whether they would vote “yes” on the scenario, we asked about the
probability that the respondent would vote in the affirmative if the referendum were to actually
arise. The referendum question was preceded by the following script:

Since you may not know for sure how you would vote on a real referendum, we are
asking you to tell us the percent chance that you would vote YES. For example, if you
check 0-10%, you are saying that you would almost surely vote NO; if you check 70-80%,
you are leaning strongly toward voting YES, but still have some doubts.

The question itself, with response categories, was:

What is the percent chance that you would vote “Yes” on the referendum to fund the Milfoil
prevention program on your lake, if the annual cost to you was $____?

☐ 0-10%    ☐ 50-60%
☐ 10-20%    ☐ 60-70%
☐ 20-30%    ☐ 70-80%
☐ 30-40%    ☐ 80-90%
☐ 40-50%    ☐ 90-100%

This question was repeated in a similarly-structured follow-up question. On the mail version of
the survey the amount of the annual cost on the follow-up was randomly assigned, whereas the
web version took advantage of the opportunity to condition the follow-up question on the
response to the initial question by lowering the annual cost if the respondent initially stated that
the probability of a “yes” vote was less than 50%, and raising it if the probability was greater
than 50%. An important point is that a follow-up question on the mail survey that would be
trivial using the conventional dichotomous yes/no approach (not to mention confusing to the
respondent)—such as asking a respondent who answered “yes” on a question where the annual
cost is $50 how he would vote if the annual cost were instead $20—provides good information in
the current context because respondents are queried about the probability of their behavior.
Finally, on the Internet survey respondents who indicated on both contingent valuation questions
that their probability of a “yes” vote was 0-10%, or who indicated on both questions that their
probability of a “yes” vote was 90-100%, were asked to state the amount that would leave their
probability of voting “yes” at “about 50%”. Figure 2 presents histograms of respondents’ stated
probability of answering “Yes” to different bid amounts for the Milfoil prevention question. The
striking aspect of figure 2 is how frequently respondents chose a probability category other than
0-10% or 90-100%, indicating substantial respondent uncertainty at all bid levels.

V. Analysis Results

We estimate two models. In the first the systematic portion of the loss function $\beta z$ is
reduced to the constant $\beta_0$ (Model 1), and in the second this expression takes the form
$\beta_0 + \beta_1 Income_j$, where $Income_j$ is the annual income of household $j$ (Model 2). We keep
the specifications simple because our focus is on identifying the average household loss $L_j$
rather than identifying the covariates that condition this loss. Overall, the estimated model
parameters include the constant $\beta_0 / \phi$, the income coefficient $\beta_1 / \phi$, the bid coefficient $1/
\phi$, the discount rate $r$, the scale ratio $\sigma / \phi$, and the random effects standard error $\theta$.

Tables 1 and 2 provide estimation results for Models 1 and 2, respectively. The
estimate of mean annual loss from a Milfoil invasion is the sample average value of $\beta z$.\footnote{Estimation generates $\frac{\beta z}{\phi}$. Dividing through by the bid coefficient $\frac{1}{\phi}$ generates $\beta z$.} The mean present value of the loss is the calculated mean annual loss multiplied by $\frac{1+r}{r}$.

Average annual willingness to pay to for the prevention program accounts for household
expectations about the probability of an invasion in the absence of the program, and, following the modeling of section 2, takes the form,

\[
\bar{WTP}^{PP}_j = \frac{P_j(1+r)}{r+P_j} \cdot \beta z_j.
\] (18)

Confidence intervals are calculated using the Krinsky-Robb (1986) method.

The results support several conclusions. First, all parameters have the expected sign, and in particular respondents with a higher household income are willing to pay more for a Milfoil prevention program. Second, the estimated mean welfare loss from a Milfoil invasion is considerably greater than the estimated mean WTP for the prevention program because most respondents believe the annual probability of an invasion is fairly low (the median annual probability is .0315). Third, although the models generate very similar values for the mean annual WTP for the prevention program (about $570), they generate very different values for the mean annual loss of an invasion. This is clearly due to differences in the estimated discount rate (9.7% versus 4.7%), as indicated in equation (6) and confirmed by a comparison of the present values of the mean welfare loss from the two models, with Model 2 generating a higher present value even though it generates a lower annual loss.

**Section VI. Convergent Validity of the Estimated Welfare Loss**

We assessed the convergent validity of the estimated welfare loss from a Milfoil invasion using two alternative analyses. The first used a contingent valuation question posed to households with shoreline property on Vilas County lakes already invaded by Milfoil. The question was presented as a referendum to control the Milfoil invasion to the point where it would be “highly unlikely” that Milfoil would cause any recreational, aesthetic, or ecological
problems on the respondent’s lake. The elicitation format was the same as for the Milfoil prevention question; in particular, we elicited the probability of a “yes” vote on the referendum. For this question there is no issue of the conflation of expectations and preferences—the welfare loss from the Milfoil invasion is immediate and certain with probability \( P_j = 1 \) — and so in estimation the term \( P_j (1 + r) / (r + P_j) \) in (12) is eliminated, and otherwise the estimation proceeds as with the Milfoil prevention case. The sample size is smaller than for the Milfoil prevention question, because fewer households in the sample reside on lakes already invaded by Milfoil, with \( N = 233 \) when household annual income is excluded as a covariate, and \( N = 198 \) when it is included.

The second analysis is the recently published hedonic study of Horsch and Lewis (2009). Using market sales data for shoreline properties in Vilas County, the authors used a difference-in-differences approach with lake-specific fixed effects to account for the possibility that unobservables correlated with a lake’s vulnerability to a Milfoil invasion might also affect the market value of its shoreline properties. The analysis examined over 1800 shoreline property sales on 172 lakes in Vilas County over the 10-year horizon 1997-2006. Table 3 presents results of the comparison. Our presumption in the table is that the present value of Milfoil loss is capitalized in shoreline property values. The Milfoil control analysis did not include estimation of the discount rate, and so for this analysis we present results for a range of discount rates.

The obvious conclusions to be drawn from this comparison are: i) the cost to shoreline property owners of a Milfoil invasion are considerable; ii) contingent valuation questions do a

---

11 It is widely understood that once in a lake Milfoil cannot be completely eradicated, and so to present a realistic scenario we chose not to claim it could be.

12 As a point of comparison, average annual income in the sample is $150,000, and the average property value in the Horsch and Lewis (2009) sample was $268,000.
reasonable job of estimating the welfare loss from an invasion; and iii) it is possible to separately identify preferences and expectations in a contingent valuation survey.

On this last point it is instructive to emphasize the consequence of overlooking or ignoring the issue of respondent expectations of a Milfoil invasion in the absence of a prevention program, and instead assuming that the estimated WTP for the Milfoil prevention program is a measure of the welfare loss from an invasion. As presented in the last section, the average value of the program is $570, far below the welfare loss obtained when expectations are accounted for. This low value arises because ignoring expectations in the econometric model is structurally equivalent to removing the term \( \frac{P_j(1+r)}{r+P_j} \) in (4), so that \( WTP=L \). This, in turn, is equivalent to setting \( P_j=1 \) for all respondents, in which case \( \frac{P_j(1+r)}{r+P_j} = 1 \). In other words, in this model ignoring expectations effectively asserts that all respondents believe that in the absence of the program an invasion is surely going to occur immediately. Such an overstatement of respondent expectations naturally leads to an underestimate of the welfare loss.

Section VII. Welfare Estimates from Re-coding Responses

As discussed in the introduction, numerous studies have examined the effect on welfare estimates of re-coding uncertainty responses as dichotomous Yes/No choices. We examined this issue using both the data for the Milfoil prevention question, and the data for the Milfoil control question. In the re-coding, choices below a specified choice probability category (e.g. 10-20%; see the referendum question above) were re-coded as “No”, and choices at the probability category and above were re-coded as “Yes”. For the Milfoil prevention question the model estimated is a logit application of equation (6):
Pr(Yes) = Pr\left(\frac{P_j(1+r)}{r+P_j} \cdot L(z_j) - t_j > \epsilon_j\right) = \frac{\exp\left(\frac{P_j(1+r) L(z_j)}{\sigma} - \frac{t_j}{\sigma}\right)}{1 + \exp\left(\frac{P_j(1+r) L(z_j)}{\sigma} - \frac{t_j}{\sigma}\right)},

where the discount rate \( r \) is fixed at its value estimated in Model 1 \((r=0.02)\), and the loss function takes the form used in Model 1, \( L(z_j) = \beta_0 \). For the Milfoil control question, the logit form is the same as this, but with the term \( \frac{P_j(1+r)}{r+P_j} \) omitted, because, as indicated in the previous section, there is no conflation of expectations and preferences concerning the presence of Milfoil. Because income was not included as an explanatory variable, sample sizes were 900 and 233 for the prevention and control models, respectively.

This model was estimated for all nine feasible re-coding possibilities. In other words, the model was estimated by using as probability boundaries for the No/Yes re-coding the levels 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90%. For example, for the model where the probability boundary was 10%, the probability choice category “0-10% was coded “No”, and the choice categories “10-20%” and higher were coded “Yes”.

Figure 3 presents the results of this analysis. As expected, as the probability boundary increases, the estimated average welfare gain from prevention/control decreases.

The average welfare gain for the model where respondent uncertainty is accounted for is $2106 for Milfoil prevention and $1226 for Milfoil control (the values in Table 1 when income is excluded as a variable). The recoding boundaries generating the closest values to these are 60% for both Milfoil prevention (welfare gain equals $2237) and Milfoil control ($1082).
Section VIII. Conclusion

Concerning the use of surveys to measure expectations, Manski (2004. pg. 42) concluded,

“Economists have long been hostile to subjective data. Caution is prudent, but hostility is not warranted…We have learned enough for me to recommend, with some confidence, that economists should abandon their antipathy to measurement of expectations. The unattractive alternative to measurement is to make unsubstantiated assumptions”.

This analysis extends this perspective to identifying the value of an environmental good in contingent valuation. To minimize complications from so-called “hypothetical bias”, contingent valuation often involves the presentation of a valuation scenario in which the respondent is asked to pay or donate to a program of environmental improvement. The WTP calculated from this scenario is often a conflation of preferences over the environmental improvement and expectations concerning the state of the environmental good in the absence of the program. To the extent that interest lies primarily in the welfare gain from the environmental improvement per se, it is incumbent to separate preferences and expectations; in other words, to understand the welfare gain associated with an environmental good, or the welfare loss associated with an environmental bad, it is critical to understand the respondent’s judgment about the outcomes in the absence of the program. One way to do this involves surveying households about their expectations, and our analysis provides theoretical and methodological particulars for the case where the environmental improvement is binary.

We are cautiously optimistic about the potential for survey questions to parse expectations and preferences, albeit with the following caveats/recommendations. First, in hindsight we believe that for binary environmental events such as exotic species invasions it would be well worth the effort and questionnaire space to ask respondents more than one question about the probability of the event in the absence of a change in management or policy.
This allows relaxing the assumption that respondents believe the annual probability of the environmental event is constant. For instance, respondents could be queried about the probability of the event over 5-year and 10-year horizons.

A second and related point is that if the environmental improvement is not binary then the issue of identifying expectations becomes more complex because the issue is no longer when an event occurs, but the evolution of the environmental change. Consider, for instance, that it is easier to estimate a reasonable model of respondent expectations about the binary state of development of a particular shoreline parcel over a 10-year horizon than it is to estimate a model of expectations about the rate of development of the entire shoreline.

Finally, we suspect that the greater the respondent’s familiarity with the environmental good in question, the more likely it is that the respondent can articulate expectations about future changes in the good, and the more important it is to allow respondents to express their subjective expectations about future changes in the good. For a mail survey, focus groups, pilot studies, and the analyst’s intuitive sense should guide the decision about whether to elicit respondent expectations. In our view the most sensible way to proceed on telephone and web surveys is to probe whether the respondent has expectations about future changes in the good, and to direct respondents with strong expectations to a series of questions to measure these expectations, and to direct other respondents to a series of statements to form their expectations. A valuable research agenda would be to examine differences in the variation in WTP between respondents with strong and weak (or no clear) expectations. We hypothesize lower variation in WTP for respondents with strong expectations (controlling for covariates), because such respondents have spent more time thinking about the good.
References


Figure 1. Histogram of Respondent Expectations of Future Milfoil Invasion
Figure 2. Histograms of Respondents’ Stated Probability of Answering “Yes” to a Bid Amount for the Milfoil prevention question.

2.1 Bid Amounts less than $100

2.2 Bid Amounts between $100 and $250

2.3 Bid Amounts between $251 and $500

2.4 Bid Amounts greater than $500
Figure 3. Estimate of Average Welfare Gain from Milfoil Prevention/Control as a Function of the Probability Threshold at Which Responses are Coded Yes vs. No.
Table 1. Estimation Results for Model 1 (Household income not included; N=900)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Rate ($r$)</td>
<td>0.098</td>
<td>0.048</td>
<td>2.022</td>
</tr>
<tr>
<td>Bid coefficient ($1/\phi$)</td>
<td>0.793</td>
<td>0.116</td>
<td>6.807</td>
</tr>
<tr>
<td>Scale ratio ($\sigma/\phi$)</td>
<td>0.669</td>
<td>0.021</td>
<td>31.725</td>
</tr>
<tr>
<td>Constant ($\beta_0/\phi$)</td>
<td>1.670</td>
<td>0.409</td>
<td>4.079</td>
</tr>
<tr>
<td>Random effects standard deviation ($\theta$)</td>
<td>1.782</td>
<td>0.385</td>
<td>4.628</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample mean estimated annual loss from a Milfoil invasion ($)</th>
<th>Mean</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2,106$</td>
<td></td>
<td>${1312, 3492}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample mean estimated present value of loss from a Milfoil invasion ($)</th>
<th>Mean</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$23,614$</td>
<td></td>
<td>${14,709, 39,145}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample mean estimated annual WTP for the prevention program ($)</th>
<th>Mean</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$563$</td>
<td></td>
<td>${351, 933}$</td>
</tr>
</tbody>
</table>
### Table 2. Estimation Results for Model 2 (Household income included; N=762)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Rate ($r$)</td>
<td>0.047</td>
<td>0.029</td>
<td>1.621</td>
</tr>
<tr>
<td>Bid coefficient ($1/\varphi$)</td>
<td>1.057</td>
<td>0.166</td>
<td>6.385</td>
</tr>
<tr>
<td>Scale ratio ($\sigma/\varphi$)</td>
<td>0.704</td>
<td>0.029</td>
<td>23.967</td>
</tr>
<tr>
<td>Constant ($\beta_0/\varphi$)</td>
<td>0.881</td>
<td>0.280</td>
<td>3.141</td>
</tr>
<tr>
<td>Income coefficient ($\beta_1/\varphi$)</td>
<td>3.828</td>
<td>0.932</td>
<td>4.105</td>
</tr>
<tr>
<td>Random effects standard deviation ($\vartheta$)</td>
<td>1.520</td>
<td>0.264</td>
<td>5.751</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample mean estimated annual loss from a Milfoil invasion ($)</th>
<th>Mean</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,373</td>
<td></td>
<td>{777, 2153}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample mean estimated present value of loss from a Milfoil invasion ($)</th>
<th>Mean</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>30,550</td>
<td></td>
<td>{17,283, 47,884}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample mean estimated annual WTP for the prevention program ($)</th>
<th>Mean</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>577</td>
<td></td>
<td>{326, 904}</td>
</tr>
</tbody>
</table>
Table 3. Comparison of Estimated Present Value of Welfare Loss from a Milfoil Invasion

<table>
<thead>
<tr>
<th>Analysis:</th>
<th>Present Value ($)</th>
<th>95% Confidence Interval ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milfoil Prevention Contingent Valuation:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>With income (estimated annual loss=$1373)</td>
<td>30,550</td>
<td>{17,283, 47,884}</td>
</tr>
<tr>
<td>Without income (estimated annual loss=$2106)</td>
<td>23,614</td>
<td>{14,709, 39,145}</td>
</tr>
<tr>
<td>Milfoil Control Contingent Valuation:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>With income (estimated annual loss=$1521):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r = 0.03 )</td>
<td>52,221</td>
<td>{40,445, 79,588}</td>
</tr>
<tr>
<td>( r = 0.06 )</td>
<td>26,871</td>
<td>{20,811, 41,375}</td>
</tr>
<tr>
<td>( r = 0.09 )</td>
<td>18,421</td>
<td>{14,267, 28,364}</td>
</tr>
<tr>
<td>( r = 0.12 )</td>
<td>14,196</td>
<td>{10,995, 21,859}</td>
</tr>
<tr>
<td>Without income (estimated annual loss=$1226):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r = 0.03 )</td>
<td>42,092</td>
<td>{33,303, 53,633}</td>
</tr>
<tr>
<td>( r = 0.06 )</td>
<td>21,659</td>
<td>{17,137, 28,426}</td>
</tr>
<tr>
<td>( r = 0.09 )</td>
<td>14,848</td>
<td>{10,778, 19,487}</td>
</tr>
<tr>
<td>( r = 0.12 )</td>
<td>11,443</td>
<td>{9,053, 15,017}</td>
</tr>
<tr>
<td>Horsch and Lewis Hedonic Study (2009):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear fixed effects</td>
<td>28,294</td>
<td>{9,656, 46,933}</td>
</tr>
<tr>
<td>Nonlinear fixed effects</td>
<td>32,087</td>
<td>{4,382, 59,792}</td>
</tr>
</tbody>
</table>
Appendix 1. Derivation of equation (11)

The observed probability of accepting the bid payment \( t_j \) has a lower bound \( \pi_{jL} \) and an upper bound \( \pi_{jH} \). Formally, we have,

\[
\pi_{jL} < \frac{\exp\left(\frac{P_j(1+r)}{r+P_j} \cdot \frac{\beta z_j + v_j}{\sigma} - \frac{t_j}{\sigma}\right)}{1 + \exp\left(\frac{P_j(1+r)}{r+P_j} \cdot \frac{\beta z_j + v_j}{\sigma} - \frac{t_j}{\sigma}\right)} < \pi_{jH} \quad (A1)
\]

It follows that the probability of the respondent choosing probability category \( C_j \), conditional on parameters \( \beta \), is the probability that \( v_j \) lies between values \( \{a, b\} \):

\[
\Pr(C_j) = \Pr(a < v_j < b | \beta) \quad . \quad (A2)
\]

Multiply both sides of (A1) by \( 1 + \exp\left(\frac{P_j(1+r)}{r+P_j} \cdot \frac{\beta z_j + v_j}{\sigma} - \frac{t_j}{\sigma}\right) \):

\[
[1 + \exp\left(\frac{P_j(1+r)}{r+P_j} \cdot \frac{\beta z_j + v_j}{\sigma} - \frac{t_j}{\sigma}\right)]\pi_{jL} < \exp\left(\frac{P_j(1+r)}{r+P_j} \cdot \frac{\beta z_j + v_j}{\sigma} - \frac{t_j}{\sigma}\right) \pi_{jH} \quad . \quad (A3)
\]

Now, multiply (A3) through by \( \exp\left(\frac{t_j}{\sigma} - \frac{P_j(1+r)}{r+P_j} \cdot \frac{\beta z_j}{\sigma}\right) \):

\[
[\exp\left(\frac{t_j}{\sigma} - \frac{P_j(1+r)}{r+P_j} \cdot \frac{\beta z_j}{\sigma}\right) + \exp\left(\frac{v_j}{\sigma}\right)]\pi_{jL} < \exp\left(\frac{v_j}{\sigma}\right) \pi_{jH} < [\exp\left(\frac{t_j}{\sigma} - \frac{P_j(1+r)}{r+P_j} \cdot \frac{\beta z_j}{\sigma}\right) + \exp\left(\frac{v_j}{\sigma}\right)]\pi_{jH} \quad . \quad (A4)
\]

Therefore,

\[
\pi_{jL}\exp\left(\frac{t_j}{\sigma} - \frac{P_j(1+r)}{r+P_j} \cdot \frac{\beta z_j}{\sigma}\right) < (1 - \pi_{jL})\exp\left(\frac{v_j}{\sigma}\right) \quad \text{and}
\]

\[
(1 - \pi_{jH})\exp\left(\frac{v_j}{\sigma}\right) < \pi_{jH}\exp\left(\frac{t_j}{\sigma} - \frac{P_j(1+r)}{r+P_j} \cdot \frac{\beta z_j}{\sigma}\right) .
\]

Therefore,

\[
\frac{\pi_{jL}}{1-\pi_{jL}} \exp\left(\frac{t_j}{\sigma} - \frac{P_j(1+r)}{r+P_j} \cdot \frac{\beta z_j}{\sigma}\right) < \exp\left(\frac{v_j}{\sigma}\right) < \frac{\pi_{jH}}{1-\pi_{jH}} \exp\left(\frac{t_j}{\sigma} - \frac{P_j(1+r)}{r+P_j} \cdot \frac{\beta z_j}{\sigma}\right) . \quad (A5)
\]

Taking logs of both sides of (A5),
\[ \ln \left[ \frac{\pi_{jL}}{(1-\pi_{jL})} \exp \left( \frac{t_j - P_j (1+r)}{r + P_j} \cdot \beta z_j \right) \right] < v_j < \ln \left[ \frac{\pi_{jH}}{(1-\pi_{jH})} \exp \left( \frac{t_j - P_j (1+r)}{r + P_j} \cdot \beta z_j \right) \right]. \]

Therefore,

\[ t_j - \frac{P_j (1+r)}{r + P_j} \cdot \beta z_j + \ln \left[ \frac{\pi_{jL}}{(1-\pi_{jL})} \right] < v_j < t_j - \frac{P_j (1+r)}{r + P_j} \cdot \beta z_j + \ln \left[ \frac{\pi_{jH}}{(1-\pi_{jH})} \right]. \quad (A6) \]

Equation (A6) is equation (11) in the text.

**Appendix 2. Derivation of equation (12)**

Considering equation (11), which identifies bounds on \( v_j \), the probability of the observed category \( C_j \) is:

\[ Pr \left( t - \frac{P_j (1+r)}{r + P_j} \cdot \beta z_j + \ln \left[ \frac{\pi_{jL}}{(1-\pi_{jL})} \right] < v_j < t - \frac{P_j (1+r)}{r + P_j} \cdot \beta z_j + \ln \left[ \frac{\pi_{jH}}{(1-\pi_{jH})} \right] \right) \]

If \( v_j \) has a logistic distribution with mean \( \mu \) and scale parameter \( \varphi \), then,

\[ Pr \left( v_j < t - \frac{P_j (1+r)}{r + P_j} \cdot \beta z_j + \ln \left[ \frac{\pi_{jL}}{(1-\pi_{jL})} \right] \right) = \frac{1}{1 + \exp \left( \frac{\mu - t - \frac{P_j (1+r)}{r + P_j} \cdot \beta z_j + \ln \left[ \frac{\pi_{jL}}{(1-\pi_{jL})} \right]}{\varphi} \right)} \]

\[ = \frac{1}{1 + \exp \left( \frac{\mu + \frac{P_j (1+r)}{r + P_j} \cdot \beta z_j - t}{\varphi} \right) \left[ \frac{1 - \pi_{jL}}{\pi_{jL}} \right]^{\frac{\sigma}{\varphi}}} \]

Therefore, since \( \mu = 0 \) when the vector \( z_j \) includes a constant, the probability of the observed response is,

\[ Pr(C_j) = \frac{1}{1 + \exp \left( \frac{\frac{P_j (1+r)}{r + P_j} \cdot \beta z_j - t}{\varphi} \right) \left[ \frac{1 - \pi_{jH}}{\pi_{jH}} \right]^{\frac{\sigma}{\varphi}}} - \frac{1}{1 + \exp \left( \frac{\frac{P_j (1+r)}{r + P_j} \cdot \beta z_j - t}{\varphi} \right) \left[ \frac{1 - \pi_{jL}}{\pi_{jL}} \right]^{\frac{\sigma}{\varphi}}} \quad (A7) \]