Using an Intervention Framework to Value Salient Ecosystem Services in a Stated Preference Experiment

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1. Introduction

Ecosystem services vary in their saliency to the people who benefit from them. Consider, for instance, the saliency of biological diversity on the one hand, and a lake’s fish population on the other. Biological diversity promotes the resiliency of natural ecosystems to external shocks in a manner that clearly benefits people (Tillman 1996; Worm et al. 2006), but this benefit is lost on most non-scientists. In contrast, a lake’s fish population is a salient ecosystem service for the recreational anglers fishing on the lake, who often have strong if not always correct beliefs about its current and future state.

To estimate the value of an improvement in the provision of an ecosystem service, analysts often use an intervention framework in a stated preference experiment. An intervention framework is defined by (i) an intervention – usually a public program to improve the provision of the ecosystem service, such as a program to conserve open space, to protect an endangered species, etc.; and (ii) the intervention effect – the difference in the provision of the ecosystem service with and without the intervention. Intervention frameworks are often favored in stated preference experiments because they create a decision environment that is sensible, if not completely familiar, to the respondent.

The contention of this paper is that analysts using an intervention framework to value an ecosystem service need to give greater consideration to the saliency of the service to the respondent population, because for salient services respondents often have prior beliefs about the intervention effect, and if these prior beliefs are different on average than implicitly assumed or explicitly presented in the valuation exercise, the estimate of the value of the improvement will be biased.
In an intervention framework for ecosystem services that are not salient to survey respondents, researcher-provided statements about the degree of provision of the ecosystem service with and without the intervention (that is, statements asserting the intervention effect) are a reasonable way—often the only way—to establish the intervention effect for the experiment. For example, the famous Exxon Valdez valuation study (Carson et al. 2005) presented survey respondents with a program to prevent future oil spills. Respondents were told that scientists believe that another oil spill in Prince William Sound of Alaska “can be expected to occur” over the next ten years. Respondents also were told that with the oil spill prevention program in place it is “virtually certain there will be no large oil spill that will affect this area”. It is reasonable to believe that respondents do not have strong prior beliefs about the likelihood of an oil spill in the absence of the prevention program, and that they do not have strong prior beliefs about the likelihood of a spill after the prevention program is put in place, and so their expectations about future oil spills with and without the prevention program are based solely on what they’re told in the survey instrument. The survey instrument, in other words, is the sole source of the respondent’s understanding of the intervention effect. Louriero and Loomis (2012) provide a more recent stated preference analysis of preventing oil spills in Europe that follows a similar pattern to the Valdez study of stating damages with and without a prevention program.

On the other hand, for the provision of salient ecosystem services the intervention effect presented in a survey instrument can be in contradiction, or at least not fully consistent, with a respondent’s prior beliefs. For example, because climate change has become politicized, statements about the effect of mitigation measures might often contradict a respondent’s prior beliefs. Lee and Cameron’s (2008) contingent valuation study of U.S. citizens’ willingness to

1 The quotes are taken from the Valdez survey, which can be found at lead author Richard Carson’s website: http://econ.ucsd.edu/~rcarson/AKsurvey.pdf.
pay to mitigate climate change examined this issue. The authors observe, “popular support for climate change mitigation policy is noticeably greater when people perceive that climate change is likely to cause greater levels of harm” (p. 246). Cai et al. (2010), Carson et al. (2010) and Viscusi and Zeckhauser (2006) also find heterogeneous expectations of climate change damages.

Failure to properly identify a respondent’s beliefs about the intervention effect implies that the analyst can only identify the average willingness to pay for the intervention itself and not for the differential provision of the ecosystem service embedded in the intervention effect. So, for instance, the analyst might ascertain the average willingness to pay for a climate change mitigation program, but without insight to respondents’ understanding of how climate will change in the absence of the program, the analyst is unable to make a claim about the value of a certain amount of climate change mitigation. In some circumstances this can be perfectly sufficient. For instance, in program benefit-cost analysis the analyst’s objective is to determine how much a respondent is willing to pay for a program; the underlying intervention effect used by respondents to arrive at their valuations might be a matter of interest, but it’s not critical to the objective. But often the analyst in interested in the value of the differential provision of the ecosystem service: the expected loss from an aquatic species invasion, the value of a particular amount of biodiversity enhancement, the benefit of mitigating a particular aspect of climate change, and so forth. In these cases the intervention is simply a vehicle to understand the value respondents place on ecosystem improvement, and so the analyst must accurately gauge the respondents understanding of the improvement provided by the intervention.

This paper expands on a recent paper by Provencher, Lewis, and Anderson (2012) – hereafter PLA – about identifying the value of a change in salient ecosystem services when the intervention effect is heterogeneous across respondents. PLA examined lakeshore property
owners’ willingness to pay for a program to prevent invasion of their lake by an aquatic invasive species (Eurasian water milfoil). In the lake-rich region examined—the northern highlands lake district of Wisconsin—such invasions are highly salient threats to lakes, and so it is sensible to assume that lakeshore property owners have strong prior beliefs about the probability of an invasion in the absence of a prevention program. It follows that if two individuals are willing to pay the same amount for the prevention program, the individual who places a higher probability on an invasion over a specified time horizon expects a greater welfare loss from an invasion. In the terminology we are using here, the prevention program is the intervention, and the intervention effect, assuming that respondents accept the assurance that the prevention program would keep the invasive species out of the lake, is the subjective probability that the lake will be invaded over the specified time horizon in the absence of the prevention program. The PLA study asked survey respondents about their subjective probabilities of an invasion over a 10-year period in the absence of a prevention program, and used these respondent probabilities and their responses to questions about their willingness to pay for the prevention program in a structural econometric model to estimate the average expected loss from an invasion.

We present two case studies to examine the use of surveys to account for respondent heterogeneity over the effect of interventions in the provision of salient ecosystem services, emphasizing that the appropriate empirical approach greatly depends on the characterization of the intervention. As in the PLA study, both case studies apply to northern Wisconsin lakes. The first—again as in the PLA study—involves a program to prevent an aquatic species invasion. We construct a structural econometric model that accounts for heterogeneity across respondents in the subjective probability of an invasion under the counterfactual (no prevention program) to estimate the welfare loss from an invasion. What distinguishes this analysis from that in the PLA
study is that, whereas the respondent sample in the PLA study is shoreline property owners, for whom the loss from an invasion is capitalized in the value of their shoreline property, in the analysis in this paper the respondent sample is composed of lake visitors—usually anglers—with the stated preference questions concerning their willingness to pay to prevent an invasion on their favorite lake. Contrasting the econometric models for the two cases is instructive, and a comparison of the expected loss of an invasion for the two groups generates good insights about the distribution of losses due to an aquatic species invasion. In particular, whereas PLA report that the average shoreline property owner is willing to pay $563 per year for a program to prevent a milfoil invasion, and would suffer an average loss of $2,106 per year from an invasion, we find that boaters are willing to pay $62 for a program to prevent a milfoil invasion on their favorite lake, and would lose an average of $97 per year from an invasion on their favorite lake.

The development of a structural model in the first case study arises because both the intervention (a program to prevent a species invasion) and the intervention effect (the reduced probability of a species invasion over a specified time horizon) are reasonably well-defined owing to the binary nature of the ecosystem output (invasion vs. no invasion). The second case applies to shoreline property owners, and concerns an intervention to change the provision of an ecosystem output that is not so easily defined: a fish restoration program that would increase fish populations by “twenty-five percent”. The difficulty in this application is that it is impossible to quantify the respondent’s understanding of the intervention effect: What exactly is meant by a 25% increase in the fish population? What is the counterfactual to the restoration program—will the population stay the same, increase anyway, or decline? And by how much? In this case, the best that can be achieved is a reduced form model in which survey questions aid the analyst in grouping respondents into subsets defined by their similarity in expressions of expected
intervention effects. This reduced-form approach has been applied to stated preference analyses of climate change (e.g. Viscusi and Zeckhauser 2006; Lee and Cameron 2008; Cai et al. 2010). We show that separately estimating WTP functions for each group of respondents generates substantially different estimates than the more parametric approach of including group-wise dummy variables in a single WTP function. Consistent with findings from climate change studies (Lee and Cameron 2008; Viscusi and Zeckhauser 2006), we find that respondents who expect no future change in their lakes’ fish populations have a lower willingness-to-pay ($99 per year) than respondents who expect modest declines in their lakes’ fish populations ($229 per year).

The rest of the paper is organized as follows. In the next section we review the literature related to the issue of properly accounting for respondent heterogeneity over intervention effects in the analysis of improvements in the provision of salient ecosystem goods. In section III we present the first case study – the prevention of milfoil on the respondent’s favorite lake of lake. The structural model developed for the analysis is similar to that used in PLA for lakeshore property owners, but whereas PLA assumed that respondents understood that the effects of invasions would be capitalized in property values, and therefore took the “long view” on the value of the prevention program – in the model we develop here we assume that, all else equal, younger respondents are willing to pay more for a prevention program than older respondents, because they expect to use the lake for more years.

Section IV presents the second case study. We conclude the presentation in section IV with several remarks about the implications of the case studies for improved survey design and good estimation of ecosystem values.
2. Literature – When should researchers worry about respondent expectations?

When does stated preference valuation of an ecosystem service face the problem of disentangling expectations from preferences? The more familiar that respondents are with the ecosystem service in question, the more likely that they will reasonably have expectations of the baseline environmental conditions that a particular hypothetical program is aimed at altering. Valuation exercises focused on private goods such as human health are likely to face situations where respondents have relevant expectations over programs in the valuation scenario. For example, Bateman and Munroe (2009) study willingness-to-pay for food that has been produced with fewer pesticides, while Rigby et al. (2009) study willingness-to-pay for food that has not been genetically modified (GM). Respondents who expect more severe baseline health risks arising from pesticides or GM are likely willing-to-pay more for foods with less of these characteristics than respondents who are less concerned. Expectations matter for valuation of many familiar private goods.

The question of whether an ecosystem service is salient to the extent that people have well-defined and heterogeneous expectations is less clear than for private goods like health. To get a sense of common themes arising in modern stated preference analyses, we reviewed papers published in *Environmental and Resource Economics* from 2009 to 2012 that use stated preference methods to value an ecosystem service that has some public goods characteristics. The two most common foci of programs used in scenarios to value ecosystem services are programs focused on the conservation of land (Mogas et al. 2009; Boyle and Ozdemir 2009; Czaajkowski and Hanley 2009; Lew and Walmo 2011; Cunha-e-Sa et al. 2012; Atkinson et al. 2012; Schuefele and Bennett 2012) and the quality or quantity of water (Jeon and Herriges 2010; Carlsson et al. 2010; Akram and Olmstead 2010; Brower et al. 2010; Kragt and Bennett 2012;
The underlying ecosystem good or service at the heart of the valuation exercises ranges from more conserved forest or farmland (Mogas et al. 2009; Boyle and Ozdemir 2009; Cunha-e-Sa et al. 2012; Schuefele and Bennett 2012), to the conservation of specific species or groups of species (e.g. Czajkowski and Hanley 2009; Carlsson et al. 2010; Lew and Walmo 2011; Atkinson et al. 2012; Kragt and Bennett 2012), to outdoor recreation (Jeon and Herriges 2010; Hess and Beharry-Borg 2012), and to water access (Brower et al. 2010; Akram and Olmstead 2010).

Whether respondent expectations are relevant for valuation programs aimed at public ecosystem services is a research judgment call. Consider the studies that ask respondents about their willingness-to-pay for a program that conserves land. If a resident of the UK is asked their WTP to conserve land in Brazil (e.g. Atkinson et al. 2012), it is probably reasonable to assume that respondents do not generally have well-formed expectations about baseline deforestation rates on the other side of the world. Conversely, if a survey respondent is asked their WTP to conserve land that is more local for them (e.g. Boyle and Ozdemir 2009; Mogas et al. 2009; Czaajkowski and Hanley 2009; Cunha-e-Sa et al. 2012; Schuefele and Bennett 2012), it becomes more likely that they have expectations of the landscape in the absence of conservation.

Figure 1 illustrates survey respondent expectations of baseline environmental changes that affect the provision of salient ecosystem services on freshwater lakes. The surveys were all conducted by authors of the present study on users of lakes in northern Wisconsin between 2005 and 2011. The figures illustrate recreational boaters’ and shoreline property owners’ expectations of an invasion of their lake by the non-native aquatic plant Eurasian Milfoil (1.a, 1.b), and property owners’ baseline expectations of their lake’s fish populations (1.c) and rate of shoreline development (1.d). Clearly these lake users – all respondents to various stated
preference surveys – do not have uniform views of baseline future ecosystem service provision on their lakes. If scope matters at all – and it should in a well-designed study – then respondents more pessimistic about future ecosystem service provision will be willing-to-pay more for programs that conserve those services than respondents who are more optimistic. The defining feature of these survey respondents is that the ecosystem services in which they are asked to value are affected by features of a local landscape in which they are familiar. Such respondents are likely to object to researcher-stated baseline conditions if they are not consistent with their own expectations. We suspect that the problem of disentangling expectations and preferences in stated preference analyses is likely to be most acute for survey populations who are asked to value programs that affect goods or regions in which they are familiar.

3. Binary change in an ecosystem service – Aquatic invasive species prevention

3.1 Model setup

In this section we focus on developing a simple model of willingness-to-pay for a program to prevent a binary negative shock to an ecosystem service. We focus on a recreational boater’s willingness-to-pay for a program to prevent an aquatic species invasion on their favorite lake – the application in this section. Let \( L(z) \) denote the annual loss from an invasion, where \( z \) denotes a vector of variables affecting boater utility, and let \( P \) denote the probability of an invasion during the year. The willingness-to-pay to assure no invasion in the initial year (year 0) is the expected loss from an invasion during year 0, \( P \cdot L(z) \). Conditional on no invasion in year 0, the willingness-to-pay to ensure no invasion in year 1 is equal to this same value discounted. The probability of no invasion in year 0 is \( 1 - P \), so the unconditional WTP to ensure no invasion in year 1 is \( \frac{1-P}{1+r} P \cdot L \). By extension, the annual WTP to ensure no invasion is:

\[
WTP(z, r) = P \cdot L + \frac{1-P}{1+r} P \cdot L + \left( \frac{1-P}{1+r} \right)^2 P \cdot L + \cdots = P \cdot L(z) \cdot \sum_{t=0}^{\infty} \left( \frac{1-P}{1+r} \right)^t
\]  

(1)
In PLA, an infinite horizon is used for (1) since the survey respondents were shoreline landowners and the invasive species is capitalized into their land values. Given the lack of such capitalization with a population of recreational boaters, an infinite horizon is not justified. We therefore specify $T$ as the respondent’s expected remaining life span. The expression in (1) can be simplified as the infinite horizon WTP minus the infinite horizon WTP discounted at year $T+1$:

$$WTP(z, r) = P \cdot L(z) \cdot \left(\frac{1+r}{r+P}\right) - \frac{P \cdot L(z)}{(1+r)^{T+1}} \cdot \left(\frac{1+r}{r+P}\right) = \left(\frac{P \cdot L(z)}{r+P}\right) \left[\frac{(1+r)^{T+1}-1}{(1+r)^T}\right]$$

(2)

The empirical challenge is identifying $L(z)$ from $P$. The approach taken in PLA, and that we adopt here, is to simply query respondents as to their subjective probabilities $P$ for the binary shock, in which case $P$ is data and is not a parameter that requires estimation. To see the importance of $P$ as data, note that if the respondent expected that the binary shock was imminent ($P=1$), then (2) reduces to a standard present value formula from $t=0$ to $t=T$ with a constant annual payment of $L(z)$. So, treating the willingness-to-pay for the prevention program as the welfare loss from the binary ecosystem shock is equivalent to assuming that respondents expect an imminent invasion. As long as $P$ is collected as respondent-specific data, equation (2) is the model that forms the basis for our econometric analysis. It is also clear from equation (2) that respondent expectations $P$ are not simply a right-hand side variable in a linear willingness-to-pay function, as has been specified in Lee and Cameron (2008) and Cai et al. (2010).

Each respondent $j$ is presented with a referendum to apply a prevention program, where respondents answer the referendum by comparing the annual payment $t_j$ to their annual WTP to ensure no invasion. Rather than allowing respondents to answer a simple yes/no to the payment

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2 The annual payment would be $rPV = L(z) \left[\frac{(1+r)^{T-1}}{(1+r)^T}\right]$, where $PV$ would be the present value of $T$ annual payments of $L(z)$. 
question, we allow respondent uncertainty and inquire as to the respondent’s probability of accepting the annual payment. While PLA also collected the respondent’s probability in terms of categories (0-10%, 10-20%, etc.), here we prompt respondents to pick a probability on a line from 0 to 1 and derive a log likelihood function that accounts for the respondents’ exact probability of voting for the referendum. Uncertainty regarding the payment is introduced by specifying their willingness-to-pay for the prevention program, \( WTP^{PP} \), as:

\[
WTP_j^{PP} = WTP_j - \varepsilon_j = \left(\frac{P_j L(z_j)}{r + P_j}\right) \left[\frac{(1+r)^{T_j+1} - 1}{(1+r)^{T_j}}\right] - \varepsilon_j
\]  

(3)

Where \( T_j \) is respondent \( j \)’s remaining lifespan. The probability of voting “yes” on the referendum is then,

\[
\pi_j = \Pr(WTP_j > t_j) = \Pr\left(\frac{P_j L(z_j)}{r + P_j} \left[\frac{(1+r)^{T_j+1} - 1}{(1+r)^{T_j}}\right] - t_j > \varepsilon_j\right)
\]  

(4)

Assuming that \( \varepsilon_j \) is logistically distributed with scale parameter \( \sigma \), we have,

\[
\pi_j = \frac{\exp \left( \frac{P_j L(z_j)}{\sigma(r+P_j)} \left[\frac{(1+r)^{T_j+1} - 1}{(1+r)^{T_j}}\right] - \frac{t_j}{\sigma} \right)}{1 + \exp \left( \frac{P_j L(z_j)}{\sigma(r+P_j)} \left[\frac{(1+r)^{T_j+1} - 1}{(1+r)^{T_j}}\right] - \frac{t_j}{\sigma} \right)}
\]  

(5)

Different respondents that face the same annual payment \( t \) and possess the same characteristics \( z \) may still arrive at different values of \( \pi_j \) due to unobserved differences between them. We account for this by expanding \( L(z_j) \) to the following linear form:

\[
L(z_j) = \beta z_j + \nu_j
\]  

(6)

Where \( \nu_j \) is an individual-specific random constant known by the respondent but not the researcher. Substituting (6) into (5) gives:
From the perspective of the analyst, $v_j$ is a random variable, and since respondents choose the probability value $\pi_j$ by picking a point along a linear line, in the case where $v_j$ is distributed logistically with scale parameter $\varphi$, this probability can be explicitly defined by appealing to the logistic probability density function. After algebraic manipulation (see appendix), the probability that the respondent selects probability $Pr_j$ is:

$$Pr_j = \frac{\exp\left[\frac{(1+\varphi)^T_j}{\sigma(1+\varphi)} \ln\left(\frac{\pi_j}{1-\pi_j}\right) + \frac{T_j}{\sigma} + \beta z_j\right]}{1 + \exp\left[\frac{(1+\varphi)^T_j}{\sigma(1+\varphi)} \ln\left(\frac{\pi_j}{1-\pi_j}\right) + \frac{T_j}{\sigma} + \beta z_j\right]}$$

The sample likelihood function is the product of the probabilities in (8) and estimated parameters include the set $\{\sigma, \beta, \varphi, r\}$.

3.2 Background and data

Eurasian watermilfoil (*Myriophyllum spicatum*) is an invasive aquatic plant that has become a major nuisance in the lake country of the northern U.S. and Canada. It is spread by boaters who inadvertently transport fragments of the plant that have become attached to their boats, anchors and trailers (Johnson, Ricciardi & Carlton, 2001). While the impact of milfoil varies considerably from lake to lake (Madsen, 1998; Smith & Barko, 1990), it has been associated with rapid growth leading to dense mats of floating vegetation that have been blamed for “clogging” infected lakes, interfering with a lake’s ecology (Boylen et al. 1999; Madsen et
al. 1991), and interfering with recreation activities (Eiswerth et al. 2005). Milfoil first invaded southern Wisconsin waters in the 1960s and spread to northern Wisconsin in the early 1990s. Empirical estimates of the welfare loss from milfoil invasions have been conducted on shoreline property owners with hedonic (Horsch and Lewis 2009) and contingent valuation analysis (Provencher et al. 2012), and range from capitalized present values of approximately $23,000 to $32,000.

The sample used for estimation was taken from participants of a trip diary program conducted during the 2011 and 2012 boating season in northern Wisconsin. Participants were recruited at public landings of 136 lakes in Vilas and Oneida Counties between Memorial Day weekend and Labor Day, as a part of a larger survey focusing on invasive species. The sampled lakes were chosen to represent variation in size, popularity, distances from population centers, and presence of invasive milfoil. Sampling was evenly divided between weekdays and weekends, and lakes were visited at different times of day throughout the study period to ensure all types of boating activities were likely to be encountered. All boating parties encountered at the boat landing were approached and the adult owner of the boat was invited to participate in the study. At the end of each boating season, a 14-page follow-up survey containing the contingent valuation question was mailed out to respondents. The survey protocol followed the tailored design method (Dillman, 2007) and participants received a payment of $25 on completion. Regular contact was maintained throughout the diary period, and five separate contacts were made regarding the follow-up survey. In all 3,004 participants were recruited into the broader study with 1,969 completions. After accounting for 39 undeliverable surveys, a final response rate for the follow-up survey of 66% was attained. Completion of the CV question was conditioned on respondents believing that their favorite lake was currently not invaded by
milfoil, a conditioning that excluded 1,081 respondents from the final estimation dataset. A further 128 respondents were excluded because they owned property on the lake.³ After filtering the remaining sample for item non-response on all required elements for our analysis, 784 individuals were retained for estimation.

3.2.1 Annual for probability of invasion

Respondents were asked for their best guess at the likelihood that their favorite lake would become invaded in the next five years:

Based on the information just provided and your previous understanding of Eurasian water milfoil (EWM) invasions, what would be your best guess of the percent chance that your favorite lake (from question 10) will become infested with EWM within the next five (5) years?

(Mark your answer by placing an X on the scale below at the appropriate spot)

A follow-up was immediately asked of respondents about their best guess of the percent chance that their favorite lake will become infested with EWM within the next ten years. The theoretical model in equation (2) depicts respondents with a constant annual probability of invasion. These five and ten year probabilities were then annualized as follows:

\[ P^*_t = 1 - (1 - P^*_t)^{T-t} \]  \hspace{1cm} (9)

where \( P^*_t \) is respondent \( j \)’s stated probability over period \( t \), in this case either \( t=5 \) or \( t=10 \). With probability responses over both a five and ten year horizon, we examined whether the two horizons generated significant differences in the annual probability of invasion. First, a linear spline was estimated that implicitly assumes respondents have a constant \( P \) over the first five years, and a different constant \( P \) from years five to ten:

³ Property owners were excluded because their willingness-to-pay a per-trip fee at the boat launch is likely confounded with their fixed costs of homeownership on the lake, and because many shoreline owners likely access the lake frequently through their property rather than the boat launch.
Second, we simply averaged $P_{j5}$ and $P_{j10}$. A simple linear regression of the spline probability on the average probability shows that the two approaches were mostly the same, with an $R^2$ of approximately 0.96. We therefore use the simple average of $P_{j5}$ and $P_{j10}$ for all remaining calculations of the annual probability of a milfoil invasion for each respondent $j$: $P_j = (P_{j5} + P_{j10})/2$. Examining invasion probability responses also revealed 77 additional problematic records which were subsequently dropped from analyses. Such records included 51 individuals who predicted $P_{j5} > P_{j10}$, and 26 individuals whose annual probability was 100%. This rendered a final sample size of 707 respondents for welfare analysis.  

### 3.2.2 Valuation question

The scenario presented to respondents describes a referendum for an invasive species prevention and early detection program that would make it “highly unlikely” that milfoil become a problem on their favorite lake. Conventional stated preference protocols were followed. Respondents were told the number of regional lakes currently infested with milfoil, the regional rate of spread of the species, and the consequences of an invasion. We consulted with local lake ecologists to construct the hypothetical program in which respondents are asked to pay, which consists of i) paid staff to monitor the lake and boats at the launch for milfoil, ii) construction of boat washing stations at the launch, and iii) boater education. A short “cheap talk” script precedes the description of the scenario and implores respondents to “think about the following referendum scenario as if you are really going to vote on it.” Consistent with the theoretical model developed

\[
P_{j10} = 1 - \frac{P_{j10} - P_{j5}}{1 - P_{j5}}
\]  

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4 Respondents had a mean age of 52 years (s.d. = 6.3 years) and were overwhelmingly male (82%) with median levels of education and household income at an associate degree and earning $65,000 to $80,000 per year. Most respondents were either employed full time (68%) or retired (25%).
in 3.1, we allow respondent uncertainty in their response to the referendum by allowing them to state the *probability* that they would vote yes:

What is the percent chance that you would vote “Yes” on the referendum to fund the program to prevent milfoil on your favorite lake, if the cost to you added $____ per trip that you visit the lake?

(Mark your answer by placing an X on the scale below at the appropriate spot)

Our use of a probability scale differs from PLA’s approach where respondents check probability categories, 0-10%, 10-20%, etc. Since the referendum would add a per-trip fee to the respondents’ daily trip cost, we also query their trip demand response by asking how their quantity of trips would compare to the most recent season. The number of trips respondents took in the most recent season was asked in a previous question. On average, respondents reduce their expected number of trips to their favorite lake by one trip per year. The demand response question provides us with two points on each respondents’ trip demand curve, which – assuming a linear demand curve – allows us to calculate their *annual* cost of the milfoil prevention program (denoted \( t_j \) above) as their reduction in consumer surplus from the referendum’s per-trip fee.

3.3 Results and sensitivity to assumed time horizon (\( T \))

Maximum likelihood was used to estimate all parameters in (8) with original code in Matlab. Estimated parameters are presented in table 1. Estimated parameters include two scale parameters, a constant, and a discount rate. Our focus is on identifying expectations separately from preferences, rather than covariates that may condition the estimated loss from a Milfoil invasion. As such, the discount rate is the only intuitive parameter estimate in table 1, and results imply that respondents discount the future at a reasonable 9%. Without any covariates on
the right-hand side, the estimate of the mean annual losses from a milfoil invasion is simply $\hat{\beta}$, the parameter on the model constant term.$^5$

An important specification question is the relevant time horizon to use for respondents. If respondents were shoreline property owners similar to the sample used in PLA, then an infinite horizon is appropriate since Milfoil is capitalized into land values (Horsch and Lewis 2009). However, our sample only includes boaters who do not own property on their favorite lake, and so the time horizon is not so obvious. We attempted to estimate $T$ as a parameter, but the highly non-linear structure of the model did not allow convergence. Therefore, we use data on survey respondents’ ages to set $T$ for respondent $j$ to be $T_j=\max(80-\text{age}_j, 5)$. This approach presumes that respondents’ time horizon is their expected remaining life span, where the age of 80 is a middle ground between the World Bank’s estimated 2013 lifespan for Canadians (81 years) and Americans (79 years). Respondents over age 75 are assumed to use a five-year time horizon.

We test the sensitivity of our estimated loss and willingness-to-pay for the prevention program to our assumptions about respondent time horizon in table 2. Major findings include the following. First, respondents’ estimated annual loss from a Milfoil invasion is about $97, which is almost forty percent larger than their estimated willingness-to-pay for the Milfoil prevention program of $62, calculated at the sample average annual invasion probability ($P=0.154$) and remaining life ($T=28.1$). The 95% confidence intervals of these two figures do not cross, providing strong evidence that willingness-to-pay estimates for programs can be far lower than loss estimates when respondents do not expect an imminent binary shock to a salient ecosystem service. Second, results are quite robust to the researcher’s assumption regarding respondents’ time horizon, as welfare estimates generally fall (increase) when the assumed time horizon

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$^5$ Estimation generates $\hat{\beta}/\hat{\phi}$. The estimated loss is calculated by dividing through by $1/\hat{\phi}$.
increases (decreases). Finally, we plot the estimated nonlinear functional relationship between willingness-to-pay for the Milfoil prevention program \(WTP^{PP}\) and the annual probability of a Milfoil invasion in Figure 2. Even if a researcher is only interested in \(WTP^{PP}\) and not in the loss function \(L(z)\), Figure 2 makes clear that our structural model of \(WTP^{PP}\) provides important information useful for transferring benefit estimates to other settings with different expectations. Figure 2 also highlights that \(WTP^{PP}\) and \(L(z)\) are more likely to diverge when respondents have low expectations of the negative binary shock.

4. Continuous change in an ecosystem service – Fish habitat enhancement

Valuing continuous changes in a salient ecosystem service presents additional challenges beyond what is found in valuing binary changes. As developed above in section 3, binary shocks present a simple case that can be exploited in a tractable structural model because expectations can be easily represented over two possible outcomes – states of the world with and without the binary shock. Rather than dealing only with expectations of two possible outcomes, continuous changes present researchers with the problem that expectations of the evolution of change must be represented, not just whether the change occurs or not. For example, Lee and Cameron’s (2008) climate change CV study inquired about respondents’ expectations of future damages over eight separate measures: agriculture/water, ecosystems, human health, oceans/weather, equity/fairness, domestic cost shares, and international cost shares. Respondents were asked to select their expectations for each of the 8 measures via a likert scale from “extremely harmed” to “extremely improved”. Expectations were included as dummy variables on the right-hand side of their estimated willingness-to-pay function.

In this section we present additional evidence of the importance of expectations over continuous changes in an ecosystem service, and evaluate the practice of introducing expectation
dummies as linear right-hand side variables that affect willingness-to-pay. The application is to a fish restoration program on freshwater lakes in the same northern Wisconsin study region that was studied above. Rather than recreational boaters, respondents are drawn from a 2008 survey of shoreline property owners and each survey respondent is presented with a program to fund aquatic habitat restoration that would increase the fish population on his or her lake by 25%. Recreational fishing is one of the primary activities that attract people to purchase shoreline property in this region, and lakeshore residents are typically well-versed in their lake’s fishery. A change in the fish population on each respondent’s lake represents a change in an extremely salient ecosystem service for this population. In this section we present a very simple idea of accounting for expectations in the case of valuing a continuous change in an ecosystem service: separately estimating willingness-to-pay functions across groups of respondent expectations. This approach is less parametric than simply including expectation measures as right-hand side variables (e.g. Viscusi and Zeckhauser 2006; Lee and Cameron 2008; Cai et al. 2010), and may be a suitable alternative when structural decomposition of expectations and preferences is challenging.

4.1. Data and issue

Data for the analysis is from a web and mail survey of a sample of lakeshore property owners in Vilas County, Wisconsin. Sample property owners were initially surveyed in the summer of 2005, with a follow-up survey administered in the early fall of 2008. The sample was drawn from shoreline property owners as identified from local tax rolls. The sampling of properties was not random, but instead favored properties on smaller lakes to assure adequate representation of such lakes, though we found no statistical effect of lake size on WTP. Overall, 2,955 households were contacted in the 2008 survey, with 1565 (53%) providing usable
responses. Not all households received a CV question on fish restoration because some households received different contingent valuation questions concerning a lake’s green frog population and prevention or control of invasive Milfoil (see PLA 2012 for results from the prevention and control Milfoil questions).

The scenario for the Fish restoration question was a lake-wide referendum for a restoration program that would increase the fish population in the respondent’s lake by 25%. The scenario design followed conventional protocols for contingent valuation. Respondents were told that the restoration program would not change the mix of fish species on their lake (bass, muskellunge, walleye, etc.), and they were told that the program would increase fish populations through habitat restoration by other property owners on their lake, and not through stocking. Respondents were told that habitat restoration activities would include placing downed logs and planting native vegetation in the lake shallows, and adding spawning gravel. Importantly, respondents were told that such habitat restoration would happen in front of other property owner’s property. We consulted with a variety of biologists in constructing the scenario for realism and included a short “cheap talk” script. As in PLA, respondents were given an annual cost of the fish restoration program and asked to pick from categories that reflect the probability they would vote for the referendum (i.e. 0-10%, 10-20%, etc.). This question was repeated in a follow-up contingent valuation question with a different annual cost. Respondents frequently chose a probability category other than 0-10% or 90-100%, indicating substantial respondent uncertainty at all bid levels.

---

6 We distinguished habitat restoration from stocking because many anglers strongly prefer wild fish to stocked fish.
7 On the mail version of the survey the amount of the annual cost on the follow-up was randomly assigned, whereas the web version lowered the annual cost if the respondent initially stated that the probability of a “yes” vote was less than 50%, and raising it if the probability was greater than 50%. On the Internet survey respondents who indicated on both contingent valuation questions that their probability of a “yes” vote was 0-10%, or who indicated on both questions that their probability of a “yes” vote was 90-100%, were also asked to state the amount that would leave their probability of voting “yes” at “about 50%”. 
Several questions preceded the CV question that will help condition responses, including a question about how the respondents would characterize their “interest in fishing” and “the fishing quality on your lake”. In addition, we probed respondents as to their baseline expectations of their lake’s fish population by asking the following question: “How would you expect the quality of fishing on your lake to change over the next 10 years?” Amongst five choices, the two most common choices amongst respondents were “no change” (571 responses) and “modest decline” (280 responses). Figure 1.c depicts responses to this question. Since we don’t have a structural decomposition of expectations and preferences like in the binary case above, we take a less parametric approach and treat answers to these two questions as separate estimations, defined over their expectations, and drop the other expectation categories due to small samples sizes.

4.2 Results

We estimate two models, which differ in whether respondents expect “no change” in their lake’s fish population or whether they expect a “modest decline”. Full development of the econometric model used in estimation is identical to PLA’s analysis of willingness-to-pay for a program to control invasive Milfoil populations on lakes where the invasive species is already present, and we refer readers to that paper for technical development. Given our separate estimations by respondent expectation level, there is no issue of conflation of expectations and preferences within each of the two expectation samples. The welfare gain from the fish restoration program is certain and so the probability term $P_j (1 + r) / (r + P_j)$ in PLA is eliminated and estimation proceeds as outlined in that analysis.

---

8 The other categories “severe decline”, “modest increase”, and “substantial increase” included sample sizes of 42, 84, and 6 respectively.
The vector of independent variables is kept simple because our focus is on identifying the average household willingness-to-pay for the fish restoration program rather than the covariates that condition this WTP. We condition WTP on the respondents’ interest in fishing (1: low, 6: high) and the respondents’ perception of the quality of fishing on their lake (1: very poor, 5: excellent). Using the notation from PLA, the estimated model parameters include the constant $\beta_0/\varphi$, the bid coefficient $1/\varphi$, the scale ratio $\sigma/\varphi$, the vector of coefficients on independent variables scaled by $\varphi$, and a random effects standard error $\theta$ representing the standard deviation of a respondent-specific unobservable that is constant across questions.

The estimation results in table 3 support several conclusions. First, respondents with greater interest in fishing are willing to pay more for the fish habitat restoration program. Second, the estimated mean welfare gain from the fish population increase is considerably greater for those respondents who expect a modest future decline in their lake’s fish population than for respondents who expect no change. The confidence intervals for the welfare gains – calculated with the Krinsky-Robb approach – do not overlap across the two samples with different baseline expectations. These general conclusions hold whether we include household income in the estimation sample or not, but we stick to a model without income to keep sample size higher.

In table 4 we also evaluate whether the WTP divergence across the two expectation samples is due to respondent characteristics, rather than the different baseline expectations. The primary result is that there appears to be no large differences in characteristics that

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9 We also checked a specification that augmented the right-hand side variables to include the household’s annual income, though this cuts the sample size from 571 to 489 (no increase sample) and from 280 to 223 (modest decline sample). WTP results are marginally – though not statistically – higher when income is included. Results are available from the authors upon request.
would likely be correlated with fishing preferences across the two samples. Therefore, it seems that the most likely explanation for the divergence in the WTP estimates across those who expect “no change” in future fishing quality and those who expect a “modest decline” is their stated expectations.

Lastly, we evaluated the practice of including a dummy variable for an expectation category (no change) as a right-hand side variable in one estimation equation with multiple expectation categories (Viscusi and Zeckhauser 2006; Lee and Cameron 2008; Cai et al. 2010), as opposed to separately estimating WTP from the sample that expected no change from the sample that expected a modest decline. Results are presented in the last column of table 3 and differ substantially from separately estimating WTP across the two samples. Separate (joint) estimation generates average WTP estimates of $99 ($14) for those who expect no change in fish populations, and $229 ($183) for those who expect a modest decline. In addition, a likelihood ratio test strongly rejects the null hypothesis that the two expectation samples can be estimated together (1% level). Our conclusion is that when facing the prospect of estimating WTP for continuous changes in a salient ecosystem service without a WTP function that allows for structural decomposition of expectations and preferences, researchers should consider the less parametric approach of separately estimating WTP across groups of respondent expectations.
References


Figure 1. Survey respondent expectations of baseline environmental changes that affect the provision of salient ecosystem services on freshwater lakes

1.a Boater’s expectations of Milfoil invasion over next ten years (n=707)

1.b Shoreline owner’s expectations of Milfoil invasion over next ten years (n=900)

1.c Shoreline owner’s expectations of fish populations over next ten years (n=983)

1.d Shoreline owner’s expectations of shoreline development over next ten years (n=1,473)

Notes: All figures are from stated preference survey respondents associated with freshwater lakes in the lakes district of northern Wisconsin, USA – Vilas and Oneida counties. 1.a is from 2010-2011 surveys of boaters, 1.b and 1.c are from a 2008 survey of shoreline property owners; 1.d is from a 2005 survey of shoreline property owners.
Figure 2. Relationship between willingness-to-pay for Milfoil prevention program and stated probability of a Milfoil invasion (sample average probability = 0.154)

Note: dashed lines represent 95% confidence intervals calculated with Krinsky-Robb method.
Table 1. Estimation results for Milfoil prevention by recreational boaters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant ( (\beta/\varphi) )</td>
<td>7.09</td>
<td>0.42</td>
<td>16.71</td>
</tr>
<tr>
<td>Discount rate ( (r) )</td>
<td>0.09</td>
<td>0.005</td>
<td>19.21</td>
</tr>
<tr>
<td>Scale parameter ( (1/\varphi) )</td>
<td>0.079</td>
<td>0.002</td>
<td>29.39</td>
</tr>
<tr>
<td>Scale ratio ( (\sigma/\varphi) )</td>
<td>0.63</td>
<td>0.12</td>
<td>5.36</td>
</tr>
</tbody>
</table>

N=707, LL=-3233.18
Table 2. Sensitivity of WTP ($) estimates to assumed respondent time horizon, where $T=\max(80\text{-age}, 5)$

<table>
<thead>
<tr>
<th></th>
<th>Mean Loss ($) from Milfoil Invasion</th>
<th>Mean WTP ($) for Milfoil Prevention Program</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Estimate</strong></td>
<td><strong>95% Confidence</strong></td>
<td><strong>Estimate</strong></td>
</tr>
<tr>
<td>$T$</td>
<td>97.31 {89.20, 105.68}</td>
<td>61.72 {56.46, 67.00}</td>
</tr>
<tr>
<td>$T+5%$</td>
<td>96.29 {88.29, 104.47}</td>
<td>61.13 {55.86, 66.44}</td>
</tr>
<tr>
<td>$T-5%$</td>
<td>98.41 {89.91, 106.76}</td>
<td>62.32 {57.04, 67.78}</td>
</tr>
<tr>
<td>$T+10%$</td>
<td>95.34 {87.22, 103.49}</td>
<td>60.56 {55.49, 65.64}</td>
</tr>
<tr>
<td>$T-10%$</td>
<td>99.61 {91.07, 108.00}</td>
<td>62.94 {57.39, 68.35}</td>
</tr>
<tr>
<td>$T+20%$</td>
<td>93.64 {85.87, 101.52}</td>
<td>59.44 {54.43, 64.40}</td>
</tr>
<tr>
<td>$T-20%$</td>
<td>102.34 {93.51, 111.28}</td>
<td>64.22 {58.48, 69.80}</td>
</tr>
<tr>
<td>$T+50%$</td>
<td>89.66 {82.48, 97.07}</td>
<td>56.38 {51.75, 60.98}</td>
</tr>
<tr>
<td>$T-50%$</td>
<td>115.16 {104.91, 125.69}</td>
<td>68.45 {61.85, 75.05}</td>
</tr>
</tbody>
</table>

*Note: all confidence intervals calculated with the Krinsky-Robb method.*
### Table 3. Estimation Results for Fish Restoration program

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No Change</th>
<th>Modest Decline</th>
<th>Both together</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant ($\beta_0/\varphi$)</td>
<td>2.10**</td>
<td>3.52**</td>
<td>1.09**</td>
</tr>
<tr>
<td></td>
<td>(0.41)</td>
<td>(0.66)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>Bid coefficient ($1/\varphi$)</td>
<td>4.79**</td>
<td>6.67**</td>
<td>1.25**</td>
</tr>
<tr>
<td></td>
<td>(0.45)</td>
<td>(0.77)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>Scale ratio ($\sigma/\varphi$)</td>
<td>0.94</td>
<td>1.12**</td>
<td>0.59**</td>
</tr>
<tr>
<td></td>
<td>(0.04)**</td>
<td>(0.07)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Fishing interest (1: low; 5: high)</td>
<td>0.46**</td>
<td>0.52**</td>
<td>0.25**</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.14)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Fishing quality (1: poor; 5: excellent)</td>
<td>0.12</td>
<td>0.21</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.17)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Random effects standard deviation ($\theta$)</td>
<td>2.40**</td>
<td>2.39**</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(0.27)</td>
<td>(0.21)</td>
</tr>
<tr>
<td>Expect no change (1:yes, 0:no)</td>
<td></td>
<td>-0.21**</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>-2456.6</td>
<td>-1276.09</td>
<td>-3871.25</td>
</tr>
<tr>
<td>N</td>
<td>571</td>
<td>280</td>
<td>851</td>
</tr>
<tr>
<td>Estimated annual WTP for the restoration program ($) – Respondents who expect “no change”</td>
<td>$99</td>
<td>-</td>
<td>$14</td>
</tr>
<tr>
<td>95% Confidence Interval</td>
<td>{54, 139}</td>
<td></td>
<td>{78, 88}</td>
</tr>
<tr>
<td>Estimated annual WTP for the restoration program ($) – Respondents who expect “modest decline”</td>
<td></td>
<td>$229</td>
<td>$183</td>
</tr>
<tr>
<td>95% Confidence Interval</td>
<td></td>
<td>{187, 270}</td>
<td>{87, 275}</td>
</tr>
</tbody>
</table>

Notes: Confidence intervals calculated with the Krinsky-Robb method.
Table 4. Average Characteristics of samples that expected a “modest decline” versus “no change” in future fishing quality on their lake

<table>
<thead>
<tr>
<th></th>
<th>Expect a modest decline in future fishing quality</th>
<th>Expect no change in future fishing quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fishing interest (1: low, 5: high)</td>
<td>3.31</td>
<td>3.14</td>
</tr>
<tr>
<td>Fishing quality (1: poor, 5: excellent)</td>
<td>3.25</td>
<td>3.43</td>
</tr>
<tr>
<td>Volunteer activities (1=yes, 0=no)</td>
<td>0.38</td>
<td>0.35</td>
</tr>
<tr>
<td>Lake Association member (1=yes, 0=no)</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>Income (1,000s $)</td>
<td>135</td>
<td>154</td>
</tr>
<tr>
<td>Lake Size (Acres)</td>
<td>98</td>
<td>104</td>
</tr>
</tbody>
</table>
Appendix 1. Derivation of Eq. (8)

We start with equation (7) in the text, and name it (A1) here:

\[
\pi_j = \frac{\exp \left( \frac{t_j}{\sigma} \left[ \frac{(1+r)^{T_j + 1} - 1}{(1+r)^{T_j}} \right] \right)}{1 + \exp \left( \frac{t_j}{\sigma} \left[ \frac{(1+r)^{T_j + 1} - 1}{(1+r)^{T_j}} \right] \right)} \tag{A1}
\]

In the survey, the respondent chooses the probability value \( \pi_j \) by picking a point along a linear line. Multiplying both sides of (A1) by \( \exp \left( \frac{t_j}{\sigma} \left[ \frac{(1+r)^{T_j + 1} - 1}{(1+r)^{T_j}} \right] \right) \) gives,

\[
\pi_j \left( 1 + \exp \left( \frac{t_j}{\sigma} \left[ \frac{(1+r)^{T_j + 1} - 1}{(1+r)^{T_j}} \right] \right) \right) = \exp \left( \frac{t_j}{\sigma} \left[ \frac{(1+r)^{T_j + 1} - 1}{(1+r)^{T_j}} \right] \right) \tag{A2}
\]

Now, multiply (A2) through by \( \exp \left( \frac{v_j}{\sigma(r + P_j)} \left[ \frac{(1+r)^{T_j + 1} - 1}{(1+r)^{T_j}} \right] \right) \) to give,

\[
\exp \left( \frac{v_j}{\sigma(r + P_j)} \left[ \frac{(1+r)^{T_j + 1} - 1}{(1+r)^{T_j}} \right] \right) = \pi_j \left( \exp \left( \frac{t_j}{\sigma} \left[ \frac{(1+r)^{T_j + 1} - 1}{(1+r)^{T_j}} \right] \right) \exp \left( \frac{v_j}{\sigma(r + P_j)} \left[ \frac{(1+r)^{T_j + 1} - 1}{(1+r)^{T_j}} \right] \right) \right) = \exp \left( \frac{v_j}{\sigma(r + P_j)} \left[ \frac{(1+r)^{T_j + 1} - 1}{(1+r)^{T_j}} \right] \right) \tag{A3}
\]

Therefore,

\[
\exp \left( \frac{t_j}{\sigma} \left[ \frac{(1+r)^{T_j + 1} - 1}{(1+r)^{T_j}} \right] \right) = \exp \left( \frac{v_j}{\sigma(r + P_j)} \left[ \frac{(1+r)^{T_j + 1} - 1}{(1+r)^{T_j}} \right] \right) \frac{(1 - \pi_j)}{\pi_j} \tag{A4}
\]

Therefore, after taking logs of both sides and isolating \( v_j \),

\[
v_j = \sigma(r + P_j) \left[ \frac{(1+r)^{T_j}}{(1+r)^{T_j + 1} - 1} \right] \ln \left\{ \frac{\pi_j}{1-\pi_j} \exp \left( \frac{t_j}{\sigma} \left[ \frac{(1+r)^{T_j + 1} - 1}{(1+r)^{T_j}} \right] \right) \right\} \tag{A5}
\]

Therefore,

\[
v_j = \sigma(r + P_j) \left[ \frac{(1+r)^{T_j}}{(1+r)^{T_j + 1} - 1} \right] \left\{ \ln \left( \frac{\pi_j}{1-\pi_j} \right) + \frac{t_j}{\sigma} \left[ \frac{(1+r)^{T_j + 1} - 1}{(1+r)^{T_j}} \right] \right\} \tag{A6}
\]

Therefore,
\[
v_j = \sigma (r + P_j) \left[ \frac{(1+r)^{T_j}}{(1+r)^{T_j+1}-1} \left[ \ln \left( \frac{\pi_j}{1-\pi_j} \right) + \frac{t_j}{\sigma} \right] - P_j \cdot (\beta z_j) \right] \quad (A7)
\]

From the perspective of the analyst, \(v_j\) is a random variable, and so the probability that respondent \(j\) chooses probability \(\pi_j\) in the referendum question is implicitly defined by the equality in (A7). In the case where \(v_j\) is distributed logistically with scale parameter \(\varphi\), this probability can be explicitly defined by appealing to the logistic pdf (not cdf as usual):

\[
\Pr(v_j = \sigma (r + P_j) \left[ \frac{(1+r)^{T_j}}{(1+r)^{T_j+1}-1} \left[ \ln \left( \frac{\pi_j}{1-\pi_j} \right) + \frac{t_j}{\sigma} \right] - P_j \cdot (\beta z_j) \right] = \frac{\exp\left[\frac{\mu - v_j}{\varphi}\right]}{\varphi\left[1 + \exp\left[\frac{\mu - v_j}{\varphi}\right]\right]^2} \quad (A8)
\]

Therefore, since \(\mu=0\) when the vector \(z_j\) includes a constant, the probability of the observed response is,

\[
\Pr(v_j = \sigma (r + P_j) \left[ \frac{(1+r)^{T_j}}{(1+r)^{T_j+1}-1} \left[ \ln \left( \frac{\pi_j}{1-\pi_j} \right) + \frac{t_j}{\sigma} \right] + P_j \cdot (\beta z_j) \right] = \frac{\exp\left[\frac{v_j}{\varphi}\right]}{\varphi\left[1 + \exp\left[\frac{v_j}{\varphi}\right]\right]^2} \quad (A9)
\]

The sample likelihood function is the product of these probabilities. Estimated parameters include the set \(\{\sigma, \beta, \varphi, r\}\). Given the highly non-linear structure with two scale parameters, an issue is what parameters can be identified in (A9)? Expand out the following:

\[
\frac{v_j}{\varphi} = \frac{1}{\varphi} \left\{ \sigma (r + P_j) \left[ \frac{(1+r)^{T_j}}{(1+r)^{T_j+1}-1} \left[ \ln \left( \frac{\pi_j}{1-\pi_j} \right) + \frac{t_j}{\sigma} \right] - P_j \cdot (\beta z_j) \right] \right\} \quad (A10)
\]

\[
= \frac{v_j}{\varphi} = \frac{\sigma}{\varphi} r \left[ \frac{(1+r)^{T_j}}{(1+r)^{T_j+1}-1} \right] \ln \left( \frac{\pi_j}{1-\pi_j} \right) + \frac{\sigma}{\varphi} \left[ \frac{(1+r)^{T_j}}{(1+r)^{T_j+1}-1} \right] P_j \cdot \ln \left( \frac{\pi_j}{1-\pi_j} \right) + r \left[ \frac{(1+r)^{T_j}}{(1+r)^{T_j+1}-1} \right] t_j + \frac{1}{\varphi} \left[ \frac{(1+r)^{T_j}}{(1+r)^{T_j+1}-1} \right] P_j \cdot t_j - \frac{\beta}{\varphi} P_j \cdot z_j \quad (A11)
\]

\[
= \frac{v_j}{\varphi} = \gamma_3 r \left[ \frac{(1+r)^{T_j}}{(1+r)^{T_j+1}-1} \right] \ln \left( \frac{\pi_j}{1-\pi_j} \right) + \gamma_3 \left[ \frac{(1+r)^{T_j}}{(1+r)^{T_j+1}-1} \right] P_j \cdot \ln \left( \frac{\pi_j}{1-\pi_j} \right) + \gamma_2 r \left[ \frac{(1+r)^{T_j}}{(1+r)^{T_j+1}-1} \right] t_j + \gamma_2 \left[ \frac{(1+r)^{T_j}}{(1+r)^{T_j+1}-1} \right] P_j \cdot t_j - \gamma_1 P_j \cdot z_j \quad (A12)
\]

Where, \(\gamma_1 = \frac{\beta}{\varphi} ; \gamma_2 = \frac{1}{\varphi} ; \gamma_3 = \frac{\sigma}{\varphi} \). So, each parameter in the set \(\{\sigma, \beta, \varphi, r\}\) can be identified.