Physics 651 Fall 2023

Homework #1

(due Friday, October 6, 2023)

1. (10 pts) Are the following sets of vectors linearly independent or dependent over the complex field?

(a) (2,-3,0), (0,0,1), (2i,i,-i)

(b) (i,1,2), (3, i,-1), (-i, 3i, 5i)

(c) (0,4,0), (i, -3i, i), (2,0,1)

2. (10 pts) Are the following sets of functions linearly independent or dependent?

(a) $2+x^2$, $3-x+4x^3$, $2x+3x^2-8x^3$

(b) $\sinh^2 x$, 1, $\cosh^2 x$

(c) x, $(x-1)^2$, $(x+1)^2$

(d) $\sin^2 x$, $\cos^2 x$, $\sin 2x$

3. (20 pts) Consider the two states $|\psi\rangle = 3i|\varphi_1\rangle + |\varphi_2\rangle$ and $|\chi\rangle = -\frac{i}{\sqrt{2}}|\varphi_1\rangle + \frac{1}{\sqrt{2}}|\varphi_2\rangle$, where $|\varphi_1\rangle, |\varphi_2\rangle$ form a complete and orthonormal basis.

   (a) Calculate $\langle\psi|\psi\rangle, \langle\chi|\psi\rangle, \langle\psi|\chi\rangle, \langle\chi|\chi\rangle$. Are the scalar products $\langle\chi|\psi\rangle$ and $\langle\psi|\chi\rangle$ equal?

   (b) Calculate $\langle\psi+\chi|\psi+\chi\rangle$.

   (c) Calculate $|\psi\rangle\langle\chi|$ and $|\chi\rangle\langle\psi|$. Are they equal?

   (d) Show that the states $|\psi\rangle$ and $|\chi\rangle$ satisfy the triangle inequality

   (e) Show that the states $|\psi\rangle$ and $|\chi\rangle$ satisfy the Schwarz inequality

4. Reading assignment: Sakurai 1.1-1.2, Nature 1999 and Nature Comm 2011 papers