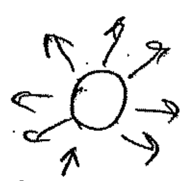
QM 651 Lecture #1

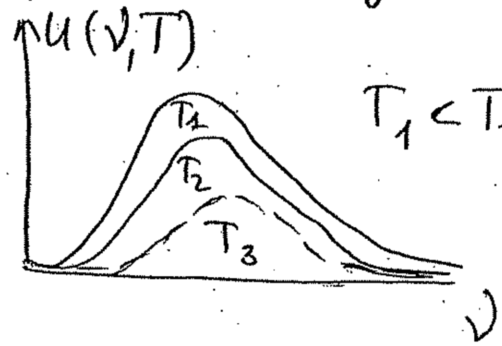
Problems with classical theory and emergence of quantum mechanics

Black-body radiation

material object that absorbs all incident radiation



heated object radiates $\Rightarrow$ spectral energy density $u(\nu, T)$



$T_1 < T_2 < T_3$

Classical description:

consider standing waves (eigenmodes) of a cavity

$\Downarrow$

solve electromagnetic wave equation with boundary conditions

$\Downarrow$

obtain $\underline{u(\nu, T) = \frac{8\pi}{c^3} \nu^2 k_B T}$ (1900)

Rayleigh – Jeans law

Problem: Blows up at $\nu \to \infty$!
(UV catastrophe)

Eventually: derivation assumed that the energy exchange between radiation and matter is <u>continuous</u>, i.e. <u>any</u> amount of energy can be exchanged. Which is WRONG!

Another approach:

Wien (1889) $\Rightarrow$ use thermodynamics and experimental Stefan-Boltzmann law $E = \sigma T^4$, $\sigma = 5{,}67 \cdot 10^{-8} \frac{W}{m^2 K^4}$

E: power emitted per unit area

σ: Stefan const

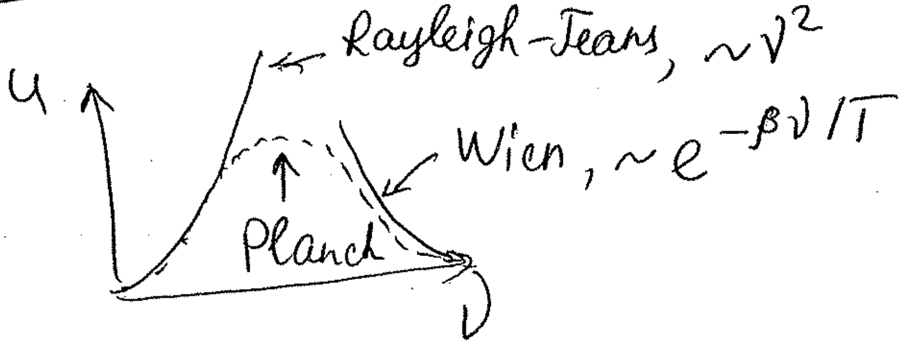
$\Downarrow$

$$u(\nu, T) = A\nu^3 e^{-\beta\nu/T}$$

where A, β are adjustable parameters

Wien's formula (1894)

Problem: fits well <u>only</u> high-frequency data



Planck (1900):

Laws of classical physics do not apply on an atomic scale

- Radiating body consists of an enormous number of elementary oscillators vibrating at all possible frequencies ν
- These oscillators are the source of the emitted radiation
- The energy of an oscillator is <u>quantized</u>

$$E = n h \nu$$

h ← Planck's constant

$$\Downarrow$$

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1}$$ ← Planck's distribution

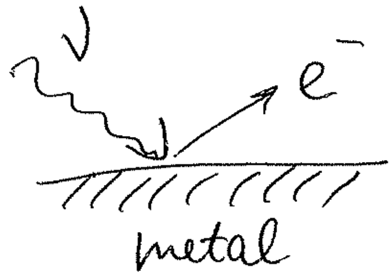
$h = 6.626 \cdot 10^{-34}$ J.s

Exact explanation of the radiation process ⇒ Einstein

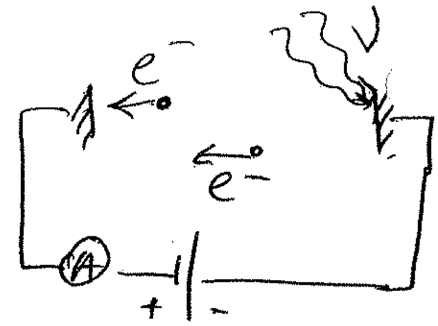
⇕

light quanta

2. Photoelectric effect



Experimental demonstration by Hertz (1887)



Experimental observations:

- monochromatic light yields electrons of a definite energy
- there is a threshold frequency ν_0 at which electron emission starts, and it's instantaneous
- At any frequency $\nu > \nu_0$, the energy of electrons is linearly proportional to the frequency of light
- Kinetic energy of the electrons depends on the frequency, but not on the intensity of the beam
- Increase in light intensity leads to the emission of more electrons, but does not change their energy

Problem: In classical physics (wave theory) $\Rightarrow$ the higher light intensity, the higher electron energy is expected

Explanation: Einstein (1905) $\Rightarrow$

$$\frac{1}{2} m v^2 = h\nu - W$$

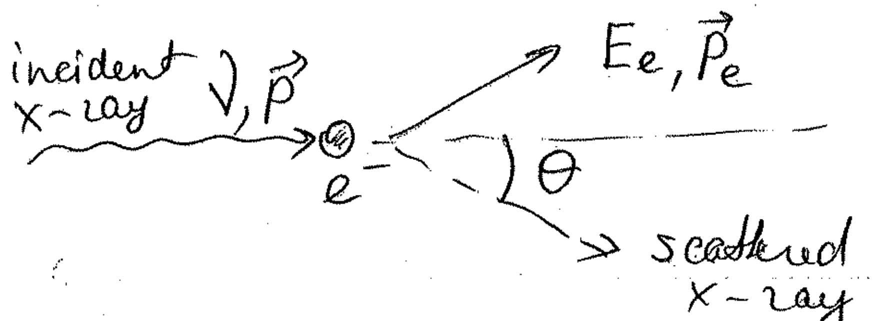
m – mass, v – velocity, W – binding energy of an electron in the metal $\Rightarrow$

$\Rightarrow$ Light radiation consists of a beam of corpuscles, the photons, of energy $h\nu$ and velocity c

($c = 3 \cdot 10^8 \frac{m}{s}$ in vacuum)

3. Compton effect

X-ray scattering by (off) free electrons



Classical prediction: • scattered light has the same frequency $\nu' = \nu$

• $I_{sc} \sim I_{inc}$

I_{sc} ← scattered intensity; I_{inc} ← incident

Experiment: $\nu \neq \nu'$, $\Delta\lambda = \lambda' - \overset{c/\nu}{\lambda} = 4\pi \lambda_c \sin^2\frac{\theta}{2}$

Compton scattering formula

$\lambda_c = \frac{\hbar}{m_e c} = 3.86 \cdot 10^{-13}\, m$ ← Compton wavelength of the electron

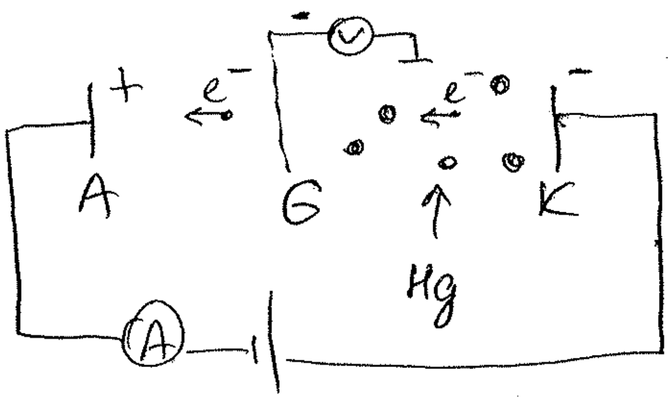
m_e ← rest mass

$\Downarrow$

Photons collide with electrons like material particles

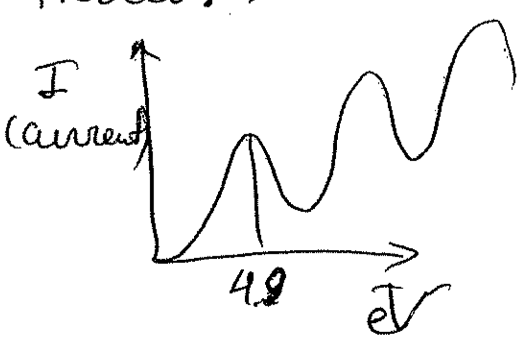
Other ground-breaking experiments:

Franck - Hertz experiment (1913)



Triode filled with Hg (mercury) vapor. Electrons are moving from K to A through a grid G, to which a small countervoltage is applied.

At electron energies below 4.9 eV the electrons don't "notice" the grid G and reach A $\Rightarrow$ the current increases as the voltage increases. At 4.9 eV,



Hg absorbs the electon's energy

$\Downarrow$

electron becomes slow and never reaches A (because of the grid G)

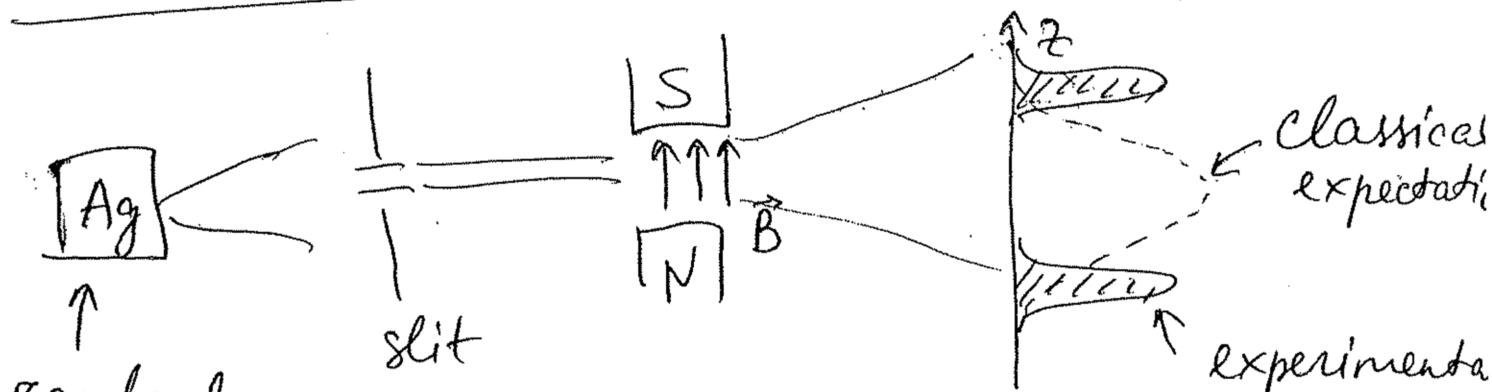
$\Downarrow$

the current drops. The situation repeats at $4.9 \times 2 = 9.8$ eV, then at $4.9 \times 3 = 14.7$ eV, etc.

$\Downarrow$

Demonstration of discreet energy levels in the Hg atom.

Stern - Gerlach experiment (1921)



randomly oriented Ag atoms (neutral) in the ground state

$\Downarrow$ Space quantization

$\Downarrow$

magnetic moment

$$\vec{\mu} = \frac{e}{m_e c}\vec{S} \Rightarrow \text{interacts with } \vec{B} \Rightarrow$$

spin

$$W = -\vec{\mu}\cdot\vec{B}$$

energy

$\Downarrow$

force along $z \Rightarrow F_z = -\frac{\partial W}{\partial z} \approx$

$$\approx \mu_z \frac{\partial B_z}{\partial z} = $$

$$= \frac{e}{m_e c} S_z \frac{\partial B_z}{\partial z}$$

Why two components?

$\Downarrow$

only two possible values of S_z

$\Downarrow$

$$S_z = \pm \frac{\hbar}{2}$$

Question: • Can this experiment be done with hydrogen atoms? If yes, in what state?
H-atoms

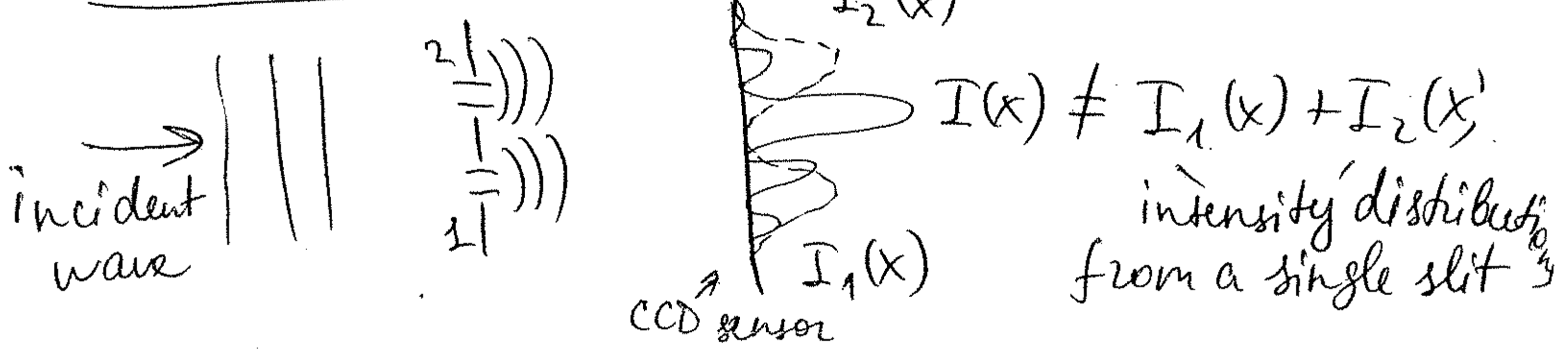
Answer: • Yes, and it was done with H-atoms later. One needs to use H-atoms in their ground state (1S), so that the angular momentum $l = 0$, otherwise $\vec{\mu} \sim (\vec{L} + \vec{S})$
$\Downarrow$
mess!

Question: Why not do this experiment with electrons?

Answer: Electrons are charged particles $\Rightarrow$ in a magnetic field the force would not be just $\frac{\partial}{\partial z}(\vec{\mu} \cdot \vec{B})$, but would also include the Lorentz force

$$\vec{F} = e \underset{\substack{\uparrow \\ \text{velocity of electron}}}{\vec{v}} \times \vec{B} \quad \Rightarrow \text{mess}$$

Young experiment



Recall classical physics:

Treat light as a conventional plain wave, with electric fields

$$E_1(\vec{r}) = E_1^0 e^{i(\vec{k}_1\cdot\vec{r}-\omega t)}$$

$$E_2(\vec{r}) = E_2^0 e^{i(\vec{k}_2\cdot\vec{r}-\omega t)}$$

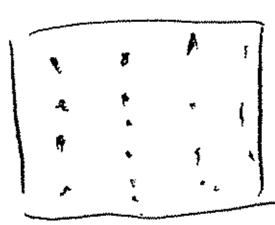
for the beam 1 and 2

Then, total $\vec{E} = \vec{E}_1 + \vec{E}_2 \Rightarrow$

intensity $I = |\vec{E}(\vec{r})|^2 = \underbrace{|E_1^0|^2}_{I_1} + \underbrace{|E_2^0|^2}_{I_2} +$

$$\underbrace{+\,2E_1^0 E_2^0 \cos(\vec{k}_1-\vec{k}_2)\cdot\vec{r}}_{\text{interference term}}$$

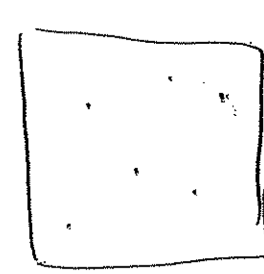
What if we reduce the amount of light, so that only one photon at a time passes through a double-slit apparatus? $\Rightarrow$ after a very long time see the same interference pattern (although we know that the photons could <u>not</u> have interacted with each other, since we let them through <u>one at a time</u>!)



CCD sensor

very long time $\Downarrow$ wave-like behavior

$\Longleftrightarrow$



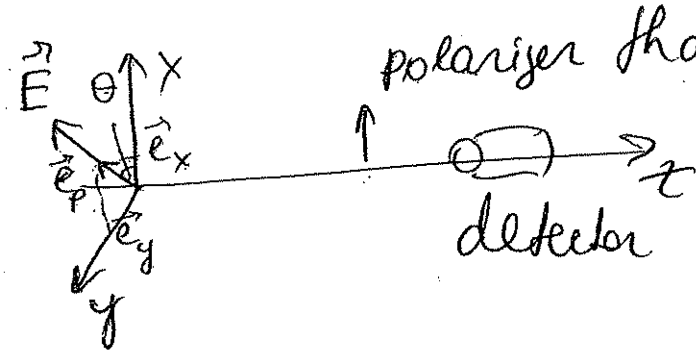
very short time (randomness) $\Downarrow$ particle-like

Conclusion: light behaves simultaneously as a wave and as a flux of particles

How do we interpret the interference pattern produced by a single photon source? $\Rightarrow$ it is a probability amplitude that the photon arrives at a given spot at a given time.

So, in QM there are no definite trajectories (in contrast to classical mechanics) $\Rightarrow$ only probability to find a system in a certain state.

Consider another example



polarizer that passes only the component polarized along X-axis

Light is characterized by

$$\vec{E}(\vec{r},t) = E_0\,\vec{e}_p\,e^{i(kz-\omega t)}$$

$\vec{e}_p, \vec{e}_x, \vec{e}_y$ - unit vectors

(propagates along z-axis)

It's linearly polarized along $\vec{e}_p$, which is under an angle θ with respect to X-axis

After the polarizer $\vec{E}'(\vec{r},t) = E_0'\,\vec{e}_x\,e^{i(kz-\omega t)}$

Light intensity $I' = |\vec{E}'|^2 = E_0'^2 = \underbrace{E_0^2}_{= I_{\text{before polarizer}}} \cos^2\theta =$

$= I_{\text{before polarizer}} \cos^2\theta$ ← Malus's law

What if $I_{\text{before polarizer}}$ is weak enough, so that the photons reach the detector one by one?! ⇒ the detector can't register "a fraction of a photon" ⇒ the photon either passes through the polarizer or does <u>not</u> pass! We do <u>not</u> know what photon will pass and which one won't → we only know the corresponding probabilities. Our detector can give only certain privileged results ⇒ <u>eigen</u> results. Each of these eigen results corresponds to an <u>eigenstate</u> ⇒ in this case we have two eigenstates: one is characterized by $\vec{e}_p = \vec{e}_x$ (pass) and another one - by $\vec{e}_p = \vec{e}_y$ (does not pass). If before the measurement the particle is in one of the eigenstates ⇒ the result of the measurement is <u>certain</u>: the detector will produce the corresponding eigenresult.

If the state before measurement is arbitrary $\Rightarrow$ only <u>probabilities</u> of obtaining different eigenresults can be predicted: $\vec{e}_p = \vec{e}_x \cos\theta + \vec{e}_y \sin\theta$

(present initial state in terms of the eigenstates)

Probability of "passing": $\cos^2\theta$, "not passing": $\sin^2\theta$

Total probability: $\underline{\cos^2\theta + \sin^2\theta = 1}$

Such decomposition in QM is called

<u>"principle of spectral decomposition"</u>

<u>Note:</u> our detector distinguishes only between the states $\vec{e}_x$ and $\vec{e}_y$ (photon detected or undetected, respectively), and info about our initial state $\vec{e}_p$ is contained <u>only</u> in probabilities to get an outcome $\vec{e}_x$ or outcome $\vec{e}_y$ $\Rightarrow$ so, the measurement event disturbs the system, "forcing" it to show only its eigenresults

This is most bizzarre and beautiful property of QM systems!!