Physics 652 Winter 2024

Homework #1

(due Wednesday, January 17, 2024)

1. (10 pts) Consider lowering and raising operators defined as follows:

   $L_+ = L_x + iL_y$; $L_- = L_x - iL_y$

   Using the commutation relations between various components of the angular momentum, calculate:

(a) $[L_z, L_+]$ and $[L_z, L_-]$;

(b) $[\mathbf{L}^2, L_+]$ and $[\mathbf{L}^2, L_-]$;

(c) Express L_-L_+ and L_+L_- in terms of $\mathbf{L}^2$ and L_z.

2. (10 pts) Based on your reading assignment concerning properties of spherical harmonics, investigate the parity of the spherical harmonics $Y_l^m(\theta,\varphi)$. In particular, how does the function $Y_l^m(\theta,\varphi)$ change under parity transformation, i.e. $\theta \rightarrow \pi - \theta$, $\varphi \rightarrow \pi + \varphi$?

3. (10 pts) Sakurai 3.24

4. Reading assignment: spherical harmonics (any text you like) + Sakurai 3.6-3.7.