MTH 252 — Lab 2
The Fundamental Theorem of Calculus

1. The graph of $f'(x)$ illustrated below is made up of line segments and two semicircles (of radius 1).

(a) Use the graph of $f'(x)$ and the Fundamental Theorem of Calculus to complete the table below. Give exact values for the missing data. And of course justify your work.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-4$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
<th>$4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Now, plot the data in the table and sketch the graph of $f(x)$. Be sure your sketch is consistent with the information you can discern from the graph of $f'(x)$.

2. The figure below shows the graph of a function $f(x)$ that has a continuous third derivative. The dashed lines are tangent to the graph of $y = f(x)$ at the points $(0, 2)$ and $(4, 1)$.

Based on what is shown, determine, if possible, whether the following integrals are positive, negative, or zero. (Hint! This lab has a title.)

(a) $\int_{0}^{4} f(x) \, dx$

(b) $\int_{0}^{4} f'(x) \, dx$

(c) $\int_{0}^{4} f''(x) \, dx$

(d) $\int_{0}^{4} f'''(x) \, dx$
3. A bicyclist is pedaling along a straight road for one hour with a velocity \( v \) shown in the figure at the right. She starts out five kilometers from the lake and positive velocities take her toward the lake. [Note: The vertical lines on the graph are at 10 minute (1/6 hour) intervals.]
(a) Does the cyclist ever turn around? If so, at what time(s)?
(b) When is she going the fastest? How fast is she going then? Toward the lake or away?
(c) When is she closest to the lake? Approximately how close to the lake does she get?
(d) When is she farthest from the lake? Approximately how far from the lake is she then?

4. The graphs in the figures below represent the velocity, \( v \), of a particle moving along the \( x \)-axis for time \( 0 \leq t \leq 5 \). The vertical scales of all graphs are the same. Identify the graph showing which particle:
(a) Has a constant acceleration.
(b) Ends up farthest to the left of where it started.
(c) Ends up the farthest from its starting point.
(d) Experiences the greatest initial acceleration.
(e) Has the greatest average velocity.
(f) Has the greatest average acceleration.

5. For the even function \( f \) graphed at the right:
(a) Suppose you know \( \int_{-2}^{2} f(x) \, dx \) and \( \int_{0}^{5} f(x) \, dx \). What is \( \int_{2}^{5} f(x) \, dx \)?
(b) Suppose you know \( \int_{-2}^{2} f(x) \, dx \) and \( \int_{-2}^{5} f(x) \, dx \). What is \( \int_{5}^{2} f(x) \, dx \)?
(c) Suppose you know \( \int_{2}^{5} f(x) \, dx \) and \( \int_{-2}^{5} f(x) \, dx \). What is \( \int_{5}^{2} f(x) \, dx \)?

6. **Food for thought!**
How does letting \( f(-4) = 4 \) in Problem 1 change the table and the graph?