The list below contains integration problems using the techniques of substitution, integration by parts, method of partial fractions, and trig substitution. Work them in the following order: 1, 2, 3, 6, 9, 10. Then work the remainder in any order.

1. Evaluate \( \int \sin^2 \theta \, d\theta \) using integration by parts letting \( u = \sin \theta \) and \( v' = \sin \theta \). Then use the identity \( \sin^2 \theta + \cos^2 \theta = 1 \) to replace \( \cos^2 \theta \). From there, you should be able to finish the problem and find the antiderivative requested.

2. Evaluate \( \int \frac{x^2}{\sqrt{9-x^2}} \, dx \) using the trig substitution \( x = 3 \sin \theta \). You will need the result from Problem 1 above to finish this. Show the triangle that fits this problem. Write your final answer in terms of \( x \).

3. Evaluate \( \int \frac{dx}{x^2\sqrt{x^2+4}} \) using the trig substitution \( x = 2 \tan \theta \). Show the triangle that fits this problem. Write your final answer in terms of \( x \).

4. Evaluate \( \int \frac{x^2}{(4-x^2)^{3/2}} \, dx \) using trig substitution. You may need to use the identity \( \sin^2 \theta + \cos^2 \theta = 1 \) to replace \( \sin^2 \theta \) in your integrand.

5. Evaluate \( \int \frac{dx}{x^2 + 16} \) using trig substitution.

6. Evaluate \( \int e^{2\theta} \sin(3\theta) \, d\theta \). You will need to do integration by parts twice.

7. Evaluate \( \int \arctan(x) \, dx \) using integration by parts.

8. Evaluate \( \int y\sqrt{y+3} \, dy \) first using the substitution \( w = y + 3 \). Then try this one using integration by parts, letting \( u = y \).

9. Evaluate \( \int \frac{1+x^2}{x(1+x)^2} \, dx \) using the method of partial fractions.

10. Evaluate \( \int \frac{2s^2 + s + 23}{(s+4)(s^2+1)} \, ds \) using the method of partial fractions.

11. You can also work Problem 1 above by rewriting the integrand using the trig identity \( \sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta) \) and evaluating without using integration by parts. Try this on your own if you like.