Section 6.3
The Sine and Cosine Functions and Their Graphs

Goals:

- Define the sine and cosine functions for any angle
- Define the sine and cosine functions for any real number
- Represent the sine and cosine functions symbolically, numerically, and graphically
- Use the sine and cosine functions in applications

Your instructor may or may not have you work on this goal; follow directions given in class:
- Model with the sine function

Vocabulary to learn:
As you read this section in your textbook, write in a definition for each of these terms. It may also help you to draw a picture to illustrate some of the terms.
Make sure you understand the meaning of each word well enough to explain it clearly to someone else and to understand how to use it in an example or homework problem.

- Periodic
- Period
- Quadrantal angles
- Unit circle
- Circular functions
- Wrapping function
- Symbolic representation
- Numerical representation
- Graphical representation
- Sinusoidal

Summary:
In this section you will explore the connections between the sine and cosine formulas you have learned to use with acute angles and formulas for use with non-acute angles. You will learn how to use the sine function and the cosine function in applications that involve any size of angle. You will use a diagram called the unit circle to help you understand and remember sine and cosine values for many different angles. You will also learn that any real number (not only angle measures) can be the input for a sine or cosine function; these real numbers are treated as if they were in radian angle measure.
You will begin to study some of the properties of the sine function and the cosine function by exploring three different representations of each function: symbolic, numerical, and graphical. Each representation will give you a different view of the function in order for you to learn more about the function and how it can be used in application. You will need to use some or all of the types of representation in each application problem you will solve.
Formulas/Procedures:

Study the relationship between the two sets of diagrams and formulas:

\[
\begin{align*}
\sin \theta &= \frac{\text{side opposite}}{\text{hypotenuse}} \\
\cos \theta &= \frac{\text{side adjacent}}{\text{hypotenuse}} \\
\sin \theta &= \frac{y}{r} \\
\cos \theta &= \frac{x}{r}
\end{align*}
\]
Study the unit circle by filling in the blanks on the next two pages. Then check your work using the solutions pages. Once you understand the unit circle, work on memorizing it.

Unit Circle

Complete the 2 formulas below to show the relationship between the sine and cosine values of an angle and the coordinates of the point where that angle’s terminal side crossed the unit circle.

Then fill in the (x,y) coordinate points for the 16 common angles shown on the unit circle.

\[
\begin{align*}
\sin \theta &= \hfill \\
\cos \theta &= \hfill 
\end{align*}
\]
<table>
<thead>
<tr>
<th>$\theta$ degrees</th>
<th>0</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>135</th>
<th>150</th>
<th>180</th>
<th>210</th>
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<th>240</th>
<th>270</th>
<th>300</th>
<th>315</th>
<th>330</th>
<th>360</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$ radians</td>
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<td>$\frac{\pi}{3}$</td>
<td>$\frac{\pi}{2}$</td>
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<td>$\frac{3\pi}{4}$</td>
<td>$\pi$</td>
<td>$\frac{5\pi}{6}$</td>
<td>$\frac{5\pi}{4}$</td>
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<td>30</td>
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<td>$\frac{5\pi}{3}$</td>
<td>$\frac{7\pi}{4}$</td>
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<td>$2\pi$</td>
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<td>2</td>
<td>3</td>
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<td>5</td>
<td>6</td>
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<td>5</td>
<td>6</td>
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<tr>
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<td>1</td>
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</tbody>
</table>
Unit Circle Handout Solutions

Complete the 2 formulas below to show the relationship between the sine and cosine values of an angle and the coordinates of the point where that angle’s terminal side crossed the unit circle.

Then fill in the (x,y) coordinate points for the 16 common angles shown on the unit circle.

\[
\sin \theta = y
\]

\[
\cos \theta = x
\]
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<tr>
<th>θ in degrees</th>
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<th>120</th>
<th>135</th>
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<th>240</th>
<th>270</th>
<th>300</th>
<th>315</th>
<th>330</th>
<th>360</th>
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</thead>
<tbody>
<tr>
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<td>π/4</td>
<td>π/3</td>
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<td>2π/3</td>
<td>5π/6</td>
<td>π</td>
<td>7π/6</td>
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<td>7π/4</td>
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<tr>
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<td>π/4</td>
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<td>Q I</td>
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<tr>
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