Sets: \( \{ A, B, C, \ldots, D \} \)

- \( n(A) \) = number of elements in \( A \).
- \( A \cup B \) = everything in \( A \) or \( B \).
- \( A \cap B \) = everything in \( A \) and also in \( B \).

\[
n(A \cup B) = n(A) + n(B) - n(A \cap B)
\]

Cocating

- Multiplication rule: If activity \( A_1 \) can be done in \( n_1 \) ways and \( A_2 \) in \( n_2 \) ways, etc. Then the sequence of activities \( A_1, A_2, \ldots, A_k \) can be done in \( n_1 \times n_2 \times \ldots \times n_k \) ways.

**Ex:** An instructor of 54 students can choose 10 students in \( \binom{54}{10} = 332,163,480 \) ways.

Roll 1 die, flip coin, draw card. How many possibilities:

\[
6 \times 2 \times 52 = 624
\]

* Note repetition

- Permutations: 

\[
P(n,k) = \text{the number of ways that } k \text{ objects, from a set of } n \text{ objects, can be arranged (order matters)}
\]

\[
P(n,k) = \frac{n!}{(n-k)!}
\]

**Ex:** How many ways can a Pres, VP and treasurer be selected from a 15 person club?

\[
P(15,3) = \frac{15!}{3! \times 12!} = 15 \times 14 \times 13
\]

\[
= 10,560,000 = 10,560
\]
Combinations:

\[ C(n,k) = \text{the number of ways } k \text{ objects can be chosen from a set of } n \text{ objects.} \]

\[ C(n,k) = \frac{n!}{(n-k)!k!} \]

Ex: How many 3 person "leadership teams" can be selected from a group of 15 people?

\[ C(15,3) = \frac{15!}{(12)!3!} \]
\[ = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2 \cdot 1} \]
\[ = 5 \cdot 7 \cdot 13 \]

Probability

Experiment: An activity that can have different outcomes (outcome(s))
Sample space: Set of all outcomes of an experiment
Event: A particular set of outcomes (a subset of the sample space)

Ex: Roll 3 coins

\[ S = \{ TTT, TTH, THT, THH, HTT, HTH, HHT, HHH \} \]
\[ E = \text{ roll 2 or more heads} \]
\[ = \{ TTH, HTH, HHT, HHH \} \quad E \subseteq S \]

Probability of an event occurring is a measure of likelihood of occurrence

\[ 0 \leq p(E) \leq 1 \]

no chance

its definite.

\* If all outcomes are equally likely (randomly chosen, etc), then

\[ p(E) = \frac{n(E)}{n(S)} \]

< only true if all outcomes are equally likely.

Ex: (from above)

\[ p(E) = \frac{4}{8} \]
\[ = \frac{1}{2} \quad (I \text{ would expect } E \text{ to occur half of the time}) \]
Note:
- If $E = \{ e_1, ..., e_k \}$ then $P(E) = P(e_1) + ... + P(e_k)$

Ex: (from above) $P(E) = P(THH) + P(HTH) + P(HTT) + P(HHH) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

- $P(\emptyset) = 0$, $P(S) = 1$

**Compound Events**

$A \cup B = A$ happens or $B$ happens

$P(A \cup B) =$ probability that $A$ happens or $B$ happens.

Ex: Probability of flipping 2 heads or 2 tails

$A \cap B = A$ happens and $B$ happens

$P(A \cap B) =$ probability of $A$ and $B$ both happening

Ex: Probability of flipping exactly 2 heads and exactly 2 tails.

$A' = A$ does not happen

$P(A') =$ the probability that $A$ will not happen

Ex: Probability of not flipping exactly 2 heads.

Formulas:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A') = 1 - P(A)$ and $P(A) = 1 - P(A')$

**Conditional Probabilities**

$P(A|B) =$ probability that $A$ will occur given that $B$ has occurred

$P(A_{\text{at least 2 heads}} | \text{1 tail}) = \frac{3}{7}$

$\{THH, HTH, HHT\}$

$\{TTT, THH, THT, TTH\}$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{1}{3} \div \frac{3}{7} = \frac{7}{9}$
Two events are independent if the occurrence of one does not affect the probability of the other.

A and B are independent if any of the following hold:

\[ P(A|B) = P(A) \]
\[ P(B|A) = P(B) \] \> definition of independence
\[ P(\text{AnB}) = P(A) \cdot P(B) \]

Note: If we know that A and B are independent, then we know only one of the following is true for independent events:

\[ P(A|B) = P(A) \]
\[ P(B|A) = P(B) \]
\[ P(\text{AnB}) = P(A) \cdot P(B) \]

Two events are mutually exclusive if the occurrence of one prevents the occurrence of the other. (They have no outcomes in common.) \( \text{AnB} = \emptyset \), so \( P(\text{AnB}) = 0 \).

Bayes' Rule: A particular application of conditional probabilities

- Partition sample space: \( S = E_1 \cup E_2 \cup \ldots \cup E_k \), \( E_i \cap E_j = \emptyset \)
- \( F \) (a different event) occurs.

Given: \( P(F|E_1) \), \( P(F|E_2) \), \ldots, \( P(F|E_k) \), \( P(E_i) \), \ldots, \( P(E_k) \)

- Asked: \( P(E_i|F) \), "Given event \( F \) happened, what is the probability you came from group \( E_i \)?"

Formula:

\[ P(E_i|F) = \frac{P(F|E_i) \cdot P(E_i)}{P(F|E_1) \cdot P(E_1) + \ldots + P(F|E_k) \cdot P(E_k)} \]
Mean, $\mu$ and $\bar{x}$
Median
Mode
Variance, $\sigma^2$ and $s^2$
St. Dev., $\sigma$ and $s$
Sample/population
Random variable.