MTH 252 — Lab 9
Applications

1. Find the total mass of the atmosphere. Assume that the atmosphere extends to +∞ and that its density, \( \rho \), is a function of height, \( h \), in meters. In particular,

\[
\rho(h) = 1.28e^{-0.000124h} \text{ kg/m}^3.
\]

The radius of the earth is approximately 6370 km.

2. Answer one of the following.
   (a) Find the center of mass of an isosceles trapezoid with height \( H \) and bases \( B_1 \) and \( B_2 \).
   (b) Find the center of mass of a cone of height \( H \) and radius \( R \).

3. The soot produced by a garbage incinerator spreads out in a circular pattern. The depth, \( H(r) \), in millimeters, of the soot deposited each month at a distance \( r \) kilometers from the incinerator is given by \( H(r) = 0.115e^{-2r} \).
   (a) Write a definite integral giving the total volume of soot deposited within 5 kilometers of the incinerator each month.
   (b) Evaluate the integral you found in part (a), giving your answer in cubic meters.

4. A metal plate, with constant density \( 2 \text{ gm/cm}^2 \), has a shape bounded by the two curves \( y = x^2 \) and \( y = \sqrt{x} \), with \( 0 \leq x \leq 1 \), and \( x, y \) in cm.
   (a) Find the total mass of the plate.
   (b) Because of the symmetry of the plate about the line \( y = x \), we have \( \bar{x} = \bar{y} \). Sketch the plate, and decide, on the basis of the shape, whether \( \bar{x} \) is less than or greater than \( 1/2 \).
   (c) Find \( \bar{x} \) and \( \bar{y} \).

5. Water is raised from a well 40 ft deep by a bucket attached to a rope. When the bucket is full, it weighs 30 lb. However, a leak in the bucket causes it to lose water at a rate of \( 1/4 \text{ lb/ft} \) for each foot that the bucket is raised. Neglecting the weight of the rope, find the work done in raising the bucket to the top.

6. A water tank is in the shape of a right circular cone with height 18 ft and radius 12 ft at the top. If it is filled with water to a depth of 15 ft, find the work done in pumping all of the water out of a spigot 2 ft above the top of the tank. (The density of water is \( \delta = 62.4 \text{ lb/ft}^3 \).)

7. An underground tank filled with gasoline of density \( 42 \text{ lb/ft}^3 \) is a hemisphere of radius 5 ft, as in the figure at the right. Use an integral to find the work to pump the gasoline over the top of the tank.