MIDTERM II
MTH 252 - Spring 2010

Name:...........................................................................................................

ID Number:...........................................................................................................

Lecture Time: 1PM 2PM

Recitation Time:..............................................................................................

Directions:

- Wait to open the test until permission is given.
- You are permitted a 3 × 5 notecard (both sides) and a non-graphing calculator.
- Make sure that you are marking the answer you intend.
- Make sure that you have completely filled out your information on both the test and the answer sheet.
- Take your time, double check your answers whenever possible.
- Grades will be posted in blackboard as soon as they are available.
- The test consists of 20 multiple choice questions worth 5 points each and 2 “write out” questions worth 10 points each for a total of 120 points.
- GOOD LUCK!!

The following formulas may be useful:

\[
\int \cos^n(x)dx = \frac{1}{n} \cos^{n-1}(x) \sin(x) + \frac{n-1}{n} \int \cos^{n-2}(x)dx, \text{ for } n \text{ a positive integer}
\]

\[
\int \sin^n(x)dx = -\frac{1}{n} \sin^{n-1}(x) \cos(x) + \frac{n-1}{n} \int \sin^{n-2}(x)dx, \text{ for } n \text{ a positive integer}
\]

\[
\int \csc^2(x)dx = -\tan(x) + c
\]

\[
\int \sec^2(x)dx = \tan(x)dx + c
\]

\[
\int \sin^2(x)dx = \frac{x}{2} - \frac{\sin(x)\cos(x)}{2} + c
\]

\[
\int \cos^2(x)dx = \frac{x}{2} + \frac{\sin(x)\cos(x)}{2} + c
\]
For Questions 1 - 20 select the correct answer. Mark your answer on your answer sheet. For problems 21 and 22 write out your answer.

1. Which method of integration would be appropriate to use for \( \int t^2 \cos(t^3) dt \)?
   a. Substitution
   b. Integration by Parts
   c. The Method of Partial Fractions
   d. A Trigonometric Substitution

   \[ \omega = t^3 \quad \frac{d\omega}{dt} = 3t^2 \]

2. Which method of integration would be appropriate to use for \( \int xe^x dx \)
   a. Substitution
   b. Integration by Parts
   c. The Method of Partial Fractions
   d. A Trigonometric Substitution

   \[ u = x \quad v' = e^x \quad u' = 1 \quad v = e^x \]

3. Which method of integration would be appropriate to use for \( \int \frac{1}{(x+1)(x-2)(x+3)} dx \)?
   a. Substitution
   b. Integration by Parts
   c. The Method of Partial Fractions
   d. A Trigonometric Substitution

   \[ \frac{1}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3} \]

4. Which method of integration would be appropriate to use for \( \int \sqrt{3x^2 - 5} dx \)?
   a. Substitution
   b. Integration by Parts
   c. The Method of Partial Fractions
   d. A Trigonometric Substitution

   \[ \omega = 3x^2 - 5 \quad d\omega = 6x \]

5. Which method of integration would be appropriate to use for \( \int \sqrt{9 - x^2} dx \)?
   a. Substitution
   b. Integration by Parts
   c. The Method of Partial Fractions
   d. A Trigonometric Substitution

   \[ \omega = 3 \quad \theta = \theta \]

   \[ \frac{3}{\sqrt{9-x^2}} \]
6. After applying the substitution \( w = x^3 + \frac{7}{3} x^2 (x^3 + 7)^{10} dx \) becomes...

a. \( \int_0^1 w^{10} dw \)

b. \( \int_2^{x^2} 3w^2 (w^3 + 1)^{10} dw \)

c. \( \frac{1}{2} \int^a_w w^{10} dw \)

d. \( \frac{1}{2} \int_a^b \mu dw \)

7. Find the area under the graph of \( f(x) = xe^x \), between \( x = 0 \) and \( x = 2 \).

a. \( \int_0^2 \left( e^x - 1 \right) dx \)

b. \( \int_0^2 \left( e^x \right) dx \)

c. \( \int_0^2 \left( e^x + 1 \right) dx \)

d. \( e^4 \)

8. When evaluating \( \int x \cos(x) \, dx \) using integration by parts, what would the correct choice of \( u \) and \( v' \) be?

a. \( u = \cos(x) \), \( v' = x \)

\( u = x \), \( v' = \cos(x) \)

b. \( u = x \), \( v' = \cos(x) \)

c. \( u = x \cos(x) \), \( v' = 1 \)

d. \( u = 1 \), \( v' = x \cos(x) \)

9. Evaluate \( \int_0^{\pi/2} x \cos(2x) dx \).

a. \( -1/2 \)

b. \( 1/2 \)

c. 0

d. 1

\( \frac{1}{x} \left[ \frac{\pi}{2} \sin(2x) + \frac{1}{2} \cos(2x) \right]_0^{\pi/2} - \frac{1}{2} \left[ -\frac{\pi}{2} \sin(2x) + \frac{1}{2} \cos(2x) \right]_0^{\pi/2} \)

\( \frac{1}{2} \sin(0) + \frac{1}{2} \cos(0) \)

\( = -\frac{1}{2}(1) \)

\( = -\frac{1}{2} \)
10. Which of the following is equal to \( \int \sin^6(x) dx \)?
   a. \( 6 \int \cos^6(x) dx \)
   b. \(-\frac{1}{6} \sin^5(x) \cos(x) - \frac{5}{6} \int \sin^4(x) dx \)
   c. \( \frac{1}{6} \cos^5(x) \sin(x) + \frac{5}{6} \int \cos^3(x) dx \)
   d. \(-\frac{1}{6} \cos^5(x) \sin(x) + \frac{5}{6} \int \sin^3(x) \cos(x) dx \)
   - From formula on page 1.

11. What would be the appropriate substitution to use to evaluate
   \[ \int t^4 \sqrt{t^5 - 1} dt \]
   a. \( w = t^5 \)
   b. \( w = t^5 - 1 \)
   c. \( w = \sqrt{t^5 + 1} \)
   d. \( w = t^4 \)

12. Applying integration by parts (once) to \( \int x \sqrt{x+1} dx \) with \( u = x \) and \( v' = \sqrt{x+1} \) yields which of the following?
   a. \( \frac{2x}{3} (x+1)^{3/2} - \frac{2}{3} \int (x+1)^{3/2} dx \)
   b. \( \frac{x}{3} (x+1)^{3/2} - \frac{2}{3} \int (x+1)^{3/2} dx \)
   c. \( x \sqrt{x+1} - \frac{2}{3} \int (x+1)^{3/2} dx \)
   d. \( \frac{2x^2}{3} (x+1)^{3/2} - \frac{2}{3} \int (x+1)^{3/2} dx \)
\[ f(x) = (x+1)^2 \]

For the problems on this page let \( f(x) = x^2 + 2x + 1 \).

13. Use Left(3) to approximate \( \int_{0}^{6} f(x) \, dx \).
   \[ \Delta x = 2 \]
   a. 70
   b. 112
   c. 118
   d. 166

14. Use Right(3) to approximate \( \int_{0}^{6} f(x) \, dx \).
   a. 70
   b. 112
   c. 118
   D 166

15. Use Mid(3) to approximate \( \int_{0}^{6} f(x) \, dx \).
   a. 70
   b. 112
   c. 118
   d. 166

16. Use Trap(3) to approximate \( \int_{0}^{6} f(x) \, dx \).

17. Suppose that we are told \( f(x) \) is continuous, \( f'(x) > 0 \) for all \( x \) and \( f''(x) > 0 \) for all \( x \). Which of the following would be true?
   a. Left(3) \( \leq \int_{0}^{1} f(x) \, dx \leq \) Right(3) and Mid(3) \( \leq \int_{0}^{1} f(x) \, dx \leq \) Trap(3).
   D Left(3) \( \leq \int_{0}^{1} f(x) \, dx \leq \) Right(3) and Mid(3) \( \geq \int_{0}^{1} f(x) \, dx \geq \) Trap(3).
   c. Left(3) \( \geq \int_{0}^{1} f(x) \, dx \geq \) Right(3) and Mid(3) \( \geq \int_{0}^{1} f(x) \, dx \geq \) Trap(3).
   d. Left(3) \( \geq \int_{0}^{1} f(x) \, dx \leq \) Right(3) and Mid(3) \( \geq \int_{0}^{1} f(x) \, dx \leq \) Trap(3).
18. What partial fractions decomposition would need to be used to evaluate
\[ \int \frac{dx}{x^3 - x^2} \]
\[ \frac{1}{x^3 - x^2} = \frac{1}{x^2(x-1)} \]
(a) \( \frac{1}{x^3} - \frac{1}{x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} \)
(b) \( \frac{1}{x^3} - \frac{1}{x} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} \)
(c) \( \frac{1}{x^3} - \frac{1}{x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} \)
(d) \( \frac{1}{x^3} - \frac{1}{x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} \)

19. Evaluate \( \int_0^1 \frac{1}{(x+1)(x-2)} \) by using the partial fractions decomposition

\[ \frac{1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2} \]

- \( x = 2 \) \quad \rightarrow \quad B = \frac{1}{3} 
- \( x = -1 \) \quad \rightarrow \quad A = -\frac{1}{3} 

\[ \int_0^1 \frac{1}{(x+1)(x-2)} \]
21. Evaluate ONE of the following two integrals. Circle the integral that you are evaluating. Show all of your work and write your final answer on the line provided.

\[ \frac{1}{2} \int \frac{e^{2x} + 0.5}{e^{2x} + x} \, dx \]

\[ \omega = e^{2x} + x \]
\[ d\omega = (2e^{2x} + 1) \, dx \]

\[ \frac{1}{2} \int \frac{d\omega}{\omega} = \frac{1}{2} \ln |e^{2x} + x| + C \]

\[ \int x^2 e^x \, dx \]
\[ u = x^2 \quad v' = e^x \]
\[ u' = 2x \quad v = e^x \]
\[ x^2 e^x - \int 2xe^x \, dx \]
\[ x^2 e^x - 2 \int xe^x \, dx \]
\[ u = x \quad v' = e^x \]
\[ x^2 e^x - 2x e^x + \int e^x \, dx \]
\[ x^2 e^x - 2x e^x - e^x + C \]
\[ \frac{1}{2} \ln |e^{2x} + x| + C \]

Solution:
22. Evaluate ONE of the following two integrals. Circle the integral that you are evaluating, show all of your work and write your final answer on the line provided.

\[
\int \frac{1}{x^2 + 2x - 15} \, dx \quad (x+5)(x-3)
\]

\[
\int \frac{x^2}{\sqrt{9-x^2}} \, dx
\]

\[
\frac{1}{(x+5)(x-3)} = \frac{A}{x+5} + \frac{B}{x-3}
\]

\[
1 = A(x-3) + B(x+5)
\]

\[
x = 3 \quad 1 = B(8) \quad B = \frac{1}{8}
\]

\[
x = -5 \quad 1 = A(-6) \quad A = -\frac{1}{8}
\]

So...

\[
\int \frac{1}{(x+5)(x-3)} \, dx = \int \frac{-\frac{1}{8}}{x+5} \, dx + \int \frac{\frac{1}{8}}{x-3} \, dx
\]

\[
= -\frac{1}{8} \ln|x+5| + \frac{1}{8} \ln|x-3| + C
\]

\[
\cos \theta = \frac{\sqrt{9-x^2}}{3}
\]

\[
3 \cos \theta = \sqrt{9-x^2}
\]

\[
\sin \theta = \frac{x}{3} \Rightarrow \theta = \arcsin \left(\frac{x}{3}\right)
\]

\[
x = 3 \sin \theta
\]

\[
dx = 3 \cos \theta \, d\theta
\]

\[
9 \int \sin^2 \theta \, d\theta = 9 \left(\frac{\theta}{2} - \frac{\sin \theta \cdot \cos \theta}{2}\right)
\]

\[
= \frac{9}{2} (\theta - \sin \theta \cdot \cos \theta) + C
\]

\[
= \frac{9}{2} \left(\arcsin \left(\frac{x}{3}\right) - \frac{x}{3} \cdot \frac{\sqrt{9-x^2}}{3}\right) + C
\]

\[
= \frac{9}{2} \arcsin \left(\frac{x}{3}\right) - \frac{x \sqrt{9-x^2}}{2} + C
\]

Solution: