MIDTERM I
MTH 252 - Spring 2010

Name: .............................................................................................................

ID Number: ........................................................................................................

Lecture Time: 1PM  2PM

Recitation Time: ............................................................................................... 

Directions:

• Wait to open the test until permission is given.
• You are permitted a 3 × 5 notecard (both sides) and a non-graphing calculator.
• Problems 1 - 21 are multiple choice and are worth 5 points each. Mark your answers on your answer sheet.
• “Write out” your answers for 22 and 23. Problem 22 is worth 5 points and problem 23 is worth 10 points.
• Make sure that you have completely filled out your information on both the test and the answer sheet.
• Take your time, double check your answers whenever possible.
• Grades will be posted in blackboard as soon as they are available.
• GOOD LUCK!!
For problems 1 - 21 choose the best answer.

1. Is \( \int_{-2}^{2} x^3 \, dx \) positive, negative or zero?
   a. \( \bigcirc \) positive
   b. negative
   c. zero
   d. It cannot be determined

2. If \( f(x) \) is an odd function and \( f(x) > 0 \) on the interval \([0,5]\), is \( \int_{-5}^{5} f(x) \, dx \) positive, negative or zero?
   a. \( \bigcirc \) positive
   b. negative
   c. zero
   d. It cannot be determined

3. Compute the average value of \( y = 2 \cos(x) \) on the interval \([0, \pi]\).
   a. \( \frac{2}{\pi} \)
   b. 4
   c. \( \frac{4}{\pi} \)
   d. \( \pi \)

4. If we wish to approximate \( \int_1^8 f(x) \, dx \) with a left sum using \( \Delta x = 2 \), which of the following would we compute?
   a. \( f(2)(3) + f(5)(3) \)
   b. \( f(2)(3) + f(4)(2) + f(6)(2) \)
   c. \( f(4)(2) + f(6)(2) + f(8)(2) \)
   d. \( f(5)(3) + f(8)(3) \)

5. If \( \int_2^5 f(x) \, dx = 2 \), determine \( \int_2^5 [2f(x) - 1] \, dx \).
   a. \( \bigcirc \) 1
   b. 2
   c. 3
   d. 4
The graph of \( f(x) \) is given below. Use the graph to answer the following questions. Each tick mark indicates one unit.

6. Determine \( \int_{1}^{2} f(x) \, dx \).
   
   a. \( \pi \)  
   b. \( \pi - 1 \)  
   c. \( \frac{\pi}{2} \)  
   d. \( \frac{\pi}{4} \)  
   
   (quarter of circle with radius 1)

7. Determine \( \int_{0}^{4} f'(x) \, dx \). Note: this is the integral of \( f'(x) \), not \( f(x) \).
   
   a. -4  
   b. -2  
   c. 2  
   d. 4  
   
   \( \int_{0}^{4} f'(x) \, dx = f(4) - f(0) = -2 - 2 = -4 \)

8. If \( F(x) \) is an antiderivative of \( f(x) \), at which of the following points does \( F(x) \) have a local maximum?
   
   a. \( x = -2 \)  
   b. \( x = 0 \)  
   c. \( x = 1 \)  
   d. \( x = 3 \)
   
   (min)  
   (inflection point)  
   (critical number, but not min/max)  
   (max)

9. If \( F(x) \) is an antiderivative of \( f(x) \) and \( F(0) = 1 \), what is \( F(1) \)?
   
   a. \( F(1) = 0 \)  
   b. \( F(1) = 1 \)  
   c. \( F(1) = 2 \)  
   d. \( F(1) = 3 \)
   
   \( \int_{0}^{1} f(x) \, dx = F(1) - F(0) \)
   
   \( \frac{1}{2} \int_{2}^{3} (x) \, dx = F(3) - F(2) \)
   
   \( 1 = F(1) - 1 \)
   
   \( 2 = F(2) \)
\[ V = \int -\sin(t) \, dt = \cos(t) - 7t + C \]
\[ V(0) = \cos(0) - 7(0) + C = 0 \]
\[ 1 + c = 0 \quad C = -1 \]
\[ v = \cos(t) - 7t - 1 \]

10. The mysterious planet X experiences fluctuations in mass, causing fluctuations in the acceleration due to gravity. The acceleration of a falling object on this planet is given by \( a = -\sin(t) - 7 \, m/s^2 \), where \( t \) is measured in seconds. If the object was dropped from a height of 100 meters, what will its position be after 2 seconds?

- 84.9 meters
- 74.9 meters
- 64.9 meters
- 54.9 meters

\[
S = \int \cos(t) - 7t - 1 \, dt = \sin(t) - 3.5t^2 - t + C
\]

\[
S(0) = \sin(0) - 3.5(0)^2 - 0 + C = 100 \quad C = 100
\]

\[
S = \sin(t) - 3.5t^2 - t + 100
\]

\[
S(2) = \sin(2) - 3.5(2)^2 - 2 + 100 = 84.9
\]

11. Kristen discovers that \( 2 \leq f(t) \leq 5 \) on the interval \([-1, 3]\). This implies which of the following statements?

- \( 2 \leq f(t) \leq 5 \) on \([-1, 3]\) implies

\[
\int_{-1}^{3} 2 \, dt \leq \int_{-1}^{3} f(t) \, dt \leq \int_{-1}^{3} 5 \, dt
\]

\[
2(3 - -1) \leq \int_{-1}^{3} f(t) \, dt \leq 5(3 - -1)
\]

\[
8 \leq \int_{-1}^{3} f(t) \, dt \leq 20
\]

For problems 12 and 13 let \( G(x) = \int_{2}^{x} 3e^t \, dt \).

12. Determine \( G(1) \).

- 0
- 3e^4
- 6e^4
- 3ln(4)

\[
G(1) = \int_{2}^{1} 3e^t \, dt = \int_{2}^{1} 3e^t \, dt = 0
\]

13. Determine \( G'(1) \).

- 0
- 3e^4
- 6e^4
- 3ln(4)

\[
G'(x) = \frac{d}{dx} \int_{2}^{x} 3e^t \, dt = \frac{d}{dx} \left[ F(2x) - F(2) \right], \text{ where } F'(t) = 3e^t
\]

\[
F'(2x)(2) - 0 = F'(2x)(2) - 0 = 3e^{2x}(2) = 3e^{4}
\]

\[
G'(1) = 6e^{4(1)^2} = 6e^4
\]
The graphs of $f(x) = x^2$ and $g(x) = -x^2 + 2$ are given below. Use the graph to answer the following questions.

14. Calculate the area of the region enclosed between the graphs of $f(x)$ and $g(x)$.
   
   a. $\frac{2}{3}x^3 + x$  
   b. $\frac{2}{3}$  
   c. $\frac{1}{3}$  
   d. $\frac{10}{3}$

   $$\int_{-1}^{1} (g(x) - f(x)) \, dx = \int_{-1}^{1} (-x^2 + 2) \, dx = \int_{-1}^{1} (-2x^2 + 2) \, dx$$

   $$= \left[ -\frac{2x^3}{3} + 2x \right]_{-1}^{1} = -\frac{2(1)^3}{3} + 2 - \left( -\frac{2(-1)^3}{3} - 2 \right)$$

   $$= \frac{8}{3}$$

15. If $h(x) = f(x) - g(x)$, is $\int_{-1}^{1} h(x) \, dx$ positive, negative or zero?
   
   a. Positive  
   b. Negative  
   c. Zero  
   d. Cannot be determined

   $$\int_{-1}^{1} h(x) \, dx = \int_{-1}^{1} (f(x) - g(x)) \, dx$$

   $$= \int_{-1}^{1} f(x) \, dx - \int_{-1}^{1} g(x) \, dx$$

   $$= (\_\_\_) - (\_\_\_)$$

16. (Not associated with the graph above) If the average value of $k(z)$ on the interval $[1, 5]$ is 4, determine $\int_{1}^{5} k(z) \, dz$.
   
   a. 4  
   b. 1  
   c. 12  
   d. 16

   Given: \( \frac{1}{5-1} \int_{1}^{5} k(z) \, dz = 4 \)

   So... \( \frac{1}{4} \int_{1}^{5} k(z) \, dz = 4 \)

   and \( \int_{1}^{5} k(z) \, dz = 16 \)
17. The graphs of $f(x)$, $f'(x)$ and $f''(x)$ are given below. Identify which graph is $f'(x)$.

- $A = f'(x)$
- $B = f'(x)$
- $C = f'(x)$
- We cannot determine which of the three graphs is $f'(x)$.

18. If the graph of $h(x)$ is given as graph A in the previous problem, which of the following integrals would be the least?

- $a. \int_{-2}^{1} h(x) \, dx$
- $b. \int_{-2}^{3} h(x) \, dx$
- $c. \int_{-2}^{1} h(x) \, dx$
- $d. \int_{-2}^{2} h(x) \, dx$

\[
\begin{align*}
\int_{-2}^{1} h(x) \, dx &< \int_{-2}^{0} h(x) \, dx < \int_{-2}^{1} h(x) \, dx \\
\text{and} \\
\int_{-2}^{1} h(x) \, dx &< \int_{-2}^{2} h(x) \, dx
\end{align*}
\]

19. TRUE or FALSE. If $\int_{a}^{b} f(x) \, dx = 0$, then it must be that $f(x) = 0$ on $[a, b]$.

- a. TRUE
- b. FALSE

Consider $\int_{-\pi}^{\pi} \sin x \, dx = 0$,

but $\sin(x)$ is not always 0.
20. If \( r(t) \) gives the rate at which a factory uses a liquid fuel source and is measured in thousands of gallons per day (and \( t \) is measured in days), what be the correct interpretation of the fact that \( \int_0^{30} r(t) \, dt = 152 \)?

- a. Over a 30 day period the company used fuel at an average rate of 152,000 gallons per day.
- □ Over a 30 day period the company used 152,000 gallons of fuel.
- c. Over a 152 day period the company used fuel at an average rate of 30,000 gallons per day.
- d. The company used a total of 30,000 gallons of fuel over a 152 day period.

21. A car traveling at 100 ft/sec (approximately 68 miles/hour) applies its brakes and begins decelerating. The car decelerates and comes to a stop in 4 seconds. Velocities for the car are recorded in the table below. Use the table to determine the maximum distance that the car could have traveled after applying the brakes?

<table>
<thead>
<tr>
<th>( t ) (seconds)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v ) (feet/second)</td>
<td>100</td>
<td>60</td>
<td>40</td>
<td>20</td>
<td>0</td>
</tr>
</tbody>
</table>

- a. 110 feet
- □ 220 feet
- c. 120 feet
- d. 170 feet

22. Solve the initial value problem. Show all of your work and write your final answer on the line provided.

\[
\frac{dy}{dx} = e^x + 2x, \quad y(0) = 4.
\]

\[
y = \int (e^x + 2x) \, dx
\]

\[
y = e^x + x^2 + c
\]

\[
y(0) = e^0 + 0^2 + c = 4
\]

\[
1 + c = 4
\]

\[
c = 3
\]

Solution: \( y = e^x + x^2 + 3 \)
23. (10 points) The graph of $f'(x)$ is given below. Sketch a graph of $f(x)$ with $f(1) = 1$. Label at least three points (in addition to the point $(1,1)$) on the graph of $f(x)$. Each tick mark indicates one unit.