Polar Coordinates and Mass/Density

MTH 252 - Lab 9

1. (a) In polar coordinates write equations for the line $x = 1$ and the circle of radius $2$ centered at the origin.

(b) Write an integral in polar coordinates representing the area to the right of $x = 1$ and inside the circle.

(c) Evaluate the integral.

2. Integrate cross-sections perpendicular to the $x$-axis to determine the area of the shape described in the previous problem. (I.e., use section 8.1 techniques to determine the area).

3. A plate occupying the region $0 \leq x \leq 2, 0 \leq y \leq 3$ has density $\delta = 5g/cm^2$.

(a) Set up two integrals giving the area of the plate, one corresponding to strips perpendicular to the $x$-axis and the other corresponding to strips perpendicular to the $y$-axis.

(b) Set up two integrals giving the total mass of the plate, one corresponding to strips perpendicular to the $x$-axis and the other corresponding to strips perpendicular to the $y$-axis.

4. The density of oil in a circular slick on the surface of the ocean at a distance $r$ meters from the center of the slick is given by $\delta(r) = \frac{50}{1+r^2} kg/m^2$. The slick extends from $r = 0$ to $r = 10,000$. Set up and evaluate an integral giving the total mass of oil in the slick.

5. Consider the region enclosed by the graphs of $y = \sqrt{x}$ and $y = x^2$.

(a) Determine the volume of the solid whose base is this region and whose cross-sections perpendicular to the $x$-axis are squares.

(b) If the density of the solid described above is given by $\delta(x) = x + 1$, determine the mass of the solid.