NAME:____ **Exercise Set #2**

MATH 627, Fall 2004

4. Let $B_\lambda = \lambda B(\lambda - B)^{-1}$, $\lambda > 0$, be the operators described in Lemma 5.3 of our text.
   (a) Show that for each $\lambda > 0$ and $\mu > 0$ we have $\lambda - B_\mu$ is *onto* the space H.
   (b) Show that each $\lambda(\lambda - B_\mu)^{-1}$ is a contraction. Thus, in the Hilbert space H, the operators $-B_\mu$, $\mu > 0$, are m-accretive.

5. If $\{S(t) : t \geq 0\}$ is a contraction semigroup with generator B, show that $\{e^{-\lambda t}S(t) :\ t \geq 0\}$ is a contraction semigroup for $\lambda > 0$, and that its generator is $B - \lambda$.

6. Let $B :\ D(B) \to H$ be linear and suppose $(\lambda - B)^{-1}$ is continuous.
   (a) Show $B(\lambda - B)^{-1} = (\lambda - B)^{-1}B$ on $D(B)$.
   (b) If $C :\ D(C) \to H$ is linear and closed, with $D(B) \subset D(C)$, show that $C(\lambda - B)^{-1}$ is closed. Note: It then follows from the *closed graph theorem* that $C(\lambda - B)^{-1}$ is continuous.