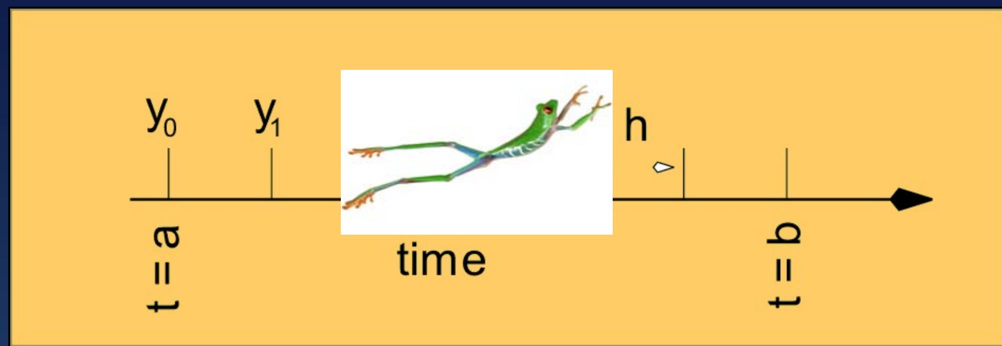
You deserve a break now!

ODE Algorithms: Lab



- How to pick *h*?
- Which is “best” ?

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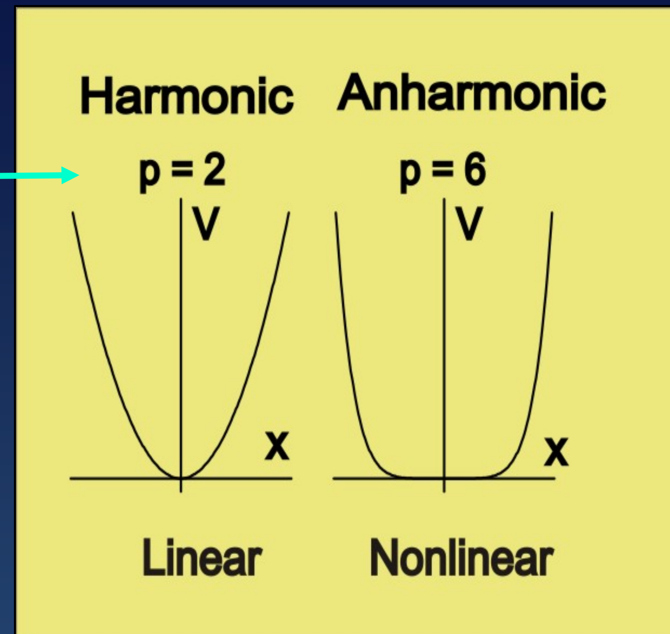
Computational Physics for Undergraduates
BS Degree Program: Oregon State University

Support by NSF & OSU

rk2, rk4 Assessment: Linear Oscillations

- **Solve ODE: *V(x)* $\propto x^p$**
- **Start: harmonic oscillator (p=2)**
- **You know answer; computer not**



$$x(t) = A\sin(\omega_0 t + \phi)$$

$$\omega_0 = 2\pi/T = \sqrt{(k/m)}$$

- **Double precision (sand!)**
- **Derivative f(t,x) = your method**
- **Pick *k*, *m* so *T* = 1**
- **Start h $\approx$ T/5; make smaller**

- **Qualitative @ 1st**
- ***y(t)* smooth, const T**
- **Plot anal & simul (no Δ)**
- **SHM: Isochronous**

  T $\neq$ T(A)

Real Assessment: Energy Conservation

- **Energy = constant/integral of motion (no friction)**
- **Not built into solution (theory ⇒)**
- **All *p* values ($V \propto x^p$)**
- **Demanding accuracy test (100, 1000's *T*)**

Metric

$$\log_{10}\left[\frac{E(t)-E(0)}{E(0)}\right]$$

$$\simeq \log_{10}\left[\frac{10^{-8}E(0)}{E(0)}\right]$$

$$\simeq -8$$

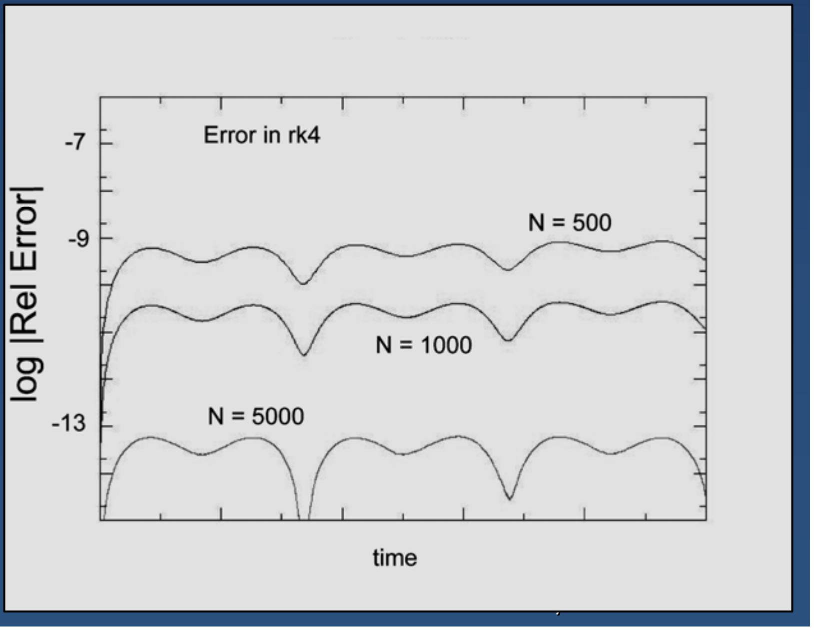
- **Plot PE, KE, total E**

$$\text{PE}(t) = V[x(t)]$$

$$\text{KE}(t) = \tfrac{1}{2}mv^2(t)$$

$$E = \text{KE}(t) + \text{PE}(t)$$

- **PE(t) ⇔ KE(t)**



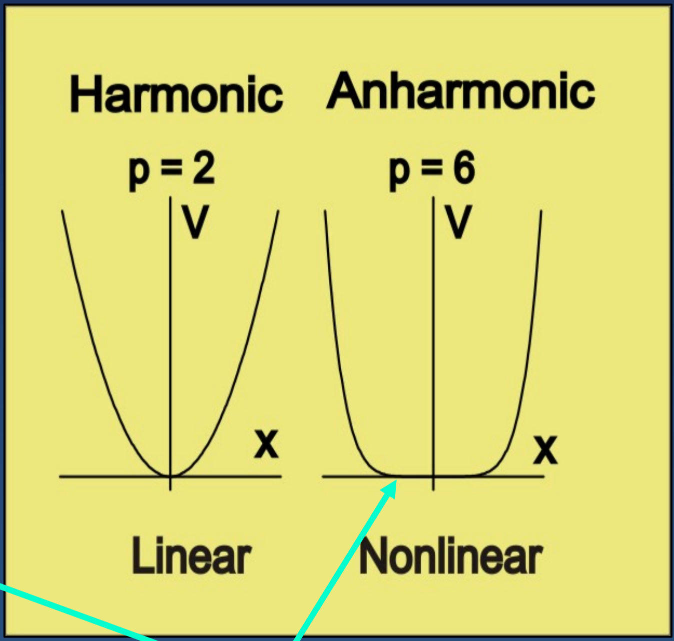
Detailed Analysis

- Decrease h for 11-15 places precision
- Fill in table

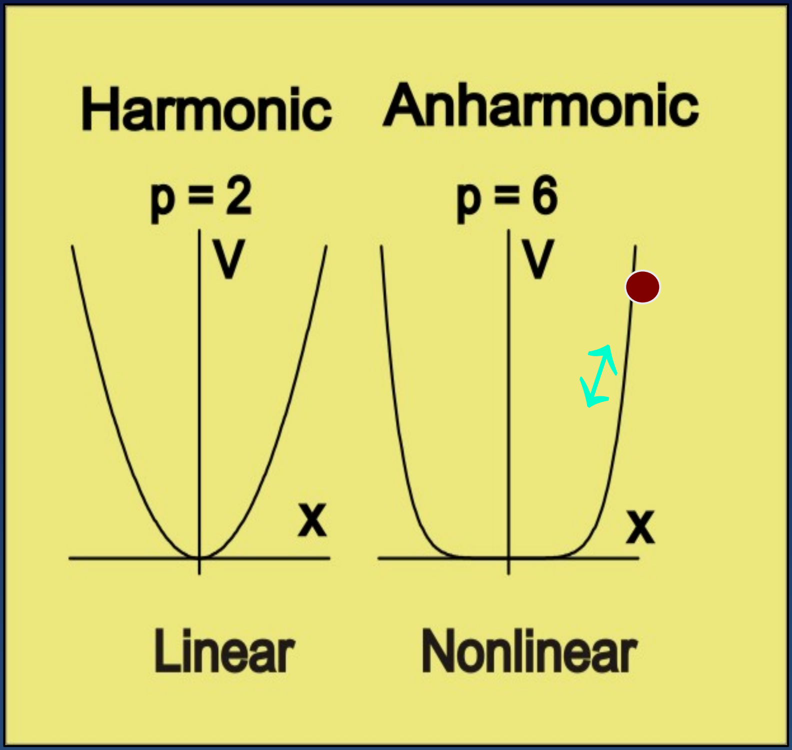
Algor	h	# flops	time	error
rk2				
rk4				
(rk45)				

- Large p $\Rightarrow$ ~ free particle
- Verify Virial theorem*

<KE> = p/2 <PE>



Model: Nonlinear Oscillator (review)



Potential: arbitrary power *p*

$$V(x) = \frac{1}{p} k x^p \quad (3$$

Force on mass

$$F_k(x) = -\frac{dV(x)}{dx} = -k x^{p-1} \quad (4$$

- **Newton's law: 2nd -*O* ODE**

$$F_{\text{ext}}(x,t) - k x^{p-1} = m \frac{d^2 x}{dt^2} \quad (5$$

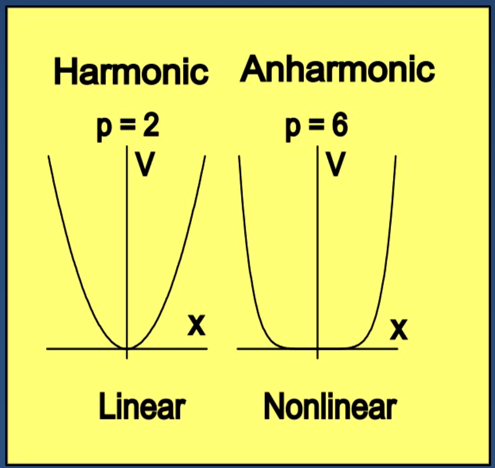
(ODE to solve)

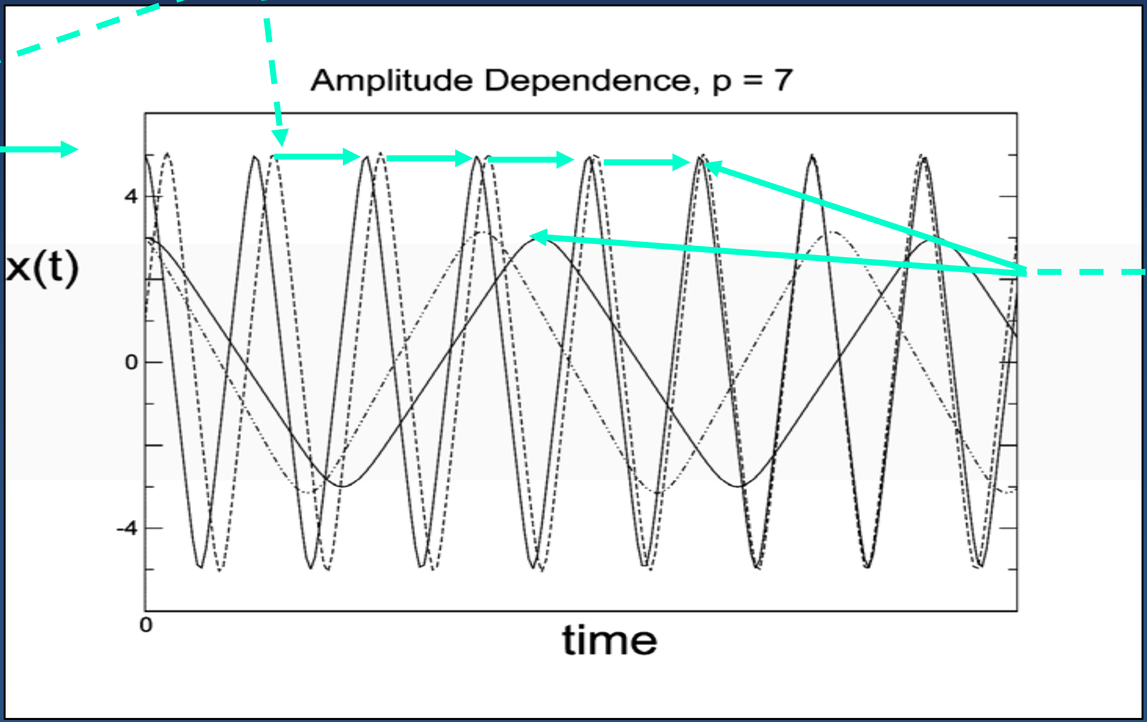
Assessment: rk4 & Nonlinear Oscillations

- rk4, ($V \propto x^p$) $4 \leq p \leq 12$
- Change h for ↑ p? (↑ accelerations)
- Fixed p & i.c.

periodic, constant A & T

- Max v at x=0, v=0 at x_{max}
- Vary i.c.: anharmonic
- Explain shape, different p





Get to Work!