Classical Chaotic Scattering

Direct ODE Application

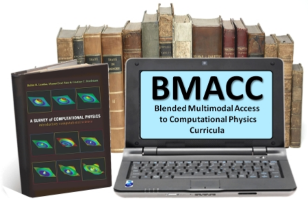
Rubin H Landau

Sally Haerer, Producer-Director

Based on *A Survey of Computational Physics* by Landau, Páez, & Bordeianu

with Support from the National Science Foundation

Course: **Computational Physics II**



What Does Classical Chaotic Scattering Look Like?

Recall Troubled Youth

- Pinball machines: multiple scattering
- Classical scattering $\Rightarrow$ continuous?
- Enough reflection $\Rightarrow$ memory loss
- Model with static potential?
- Need active bumpers?

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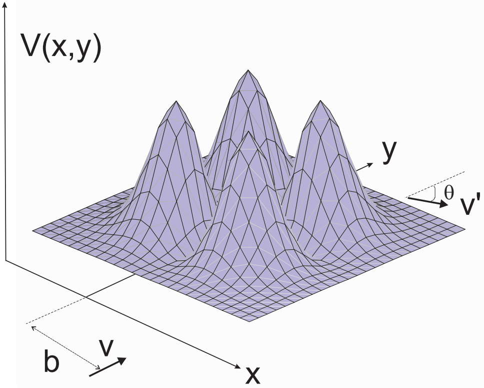
Model and Theory

Static 2-D Potential $V(x, y) = \pm\ x^2 y^2 e^{-(x^2+y^2)}$



- Scatter (high energy)
- $\pm$: repulsive/attractive
- 4 peaks $\Rightarrow$ internal reflections?
- **Theory**: Classical Dynamics
- $\mathbf{F} = m\mathbf{a}$

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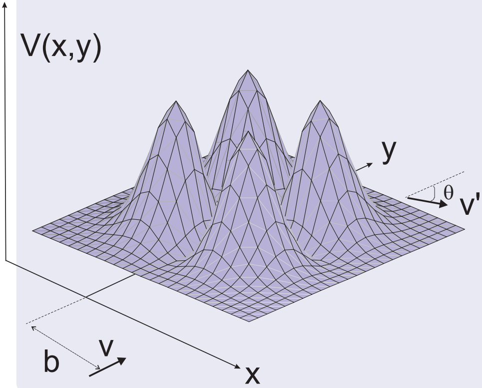

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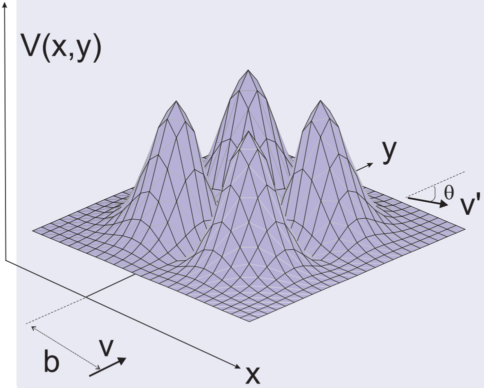

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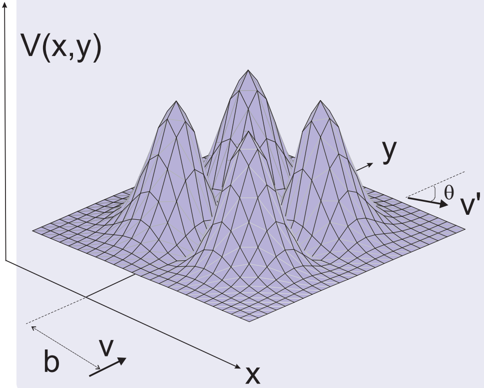

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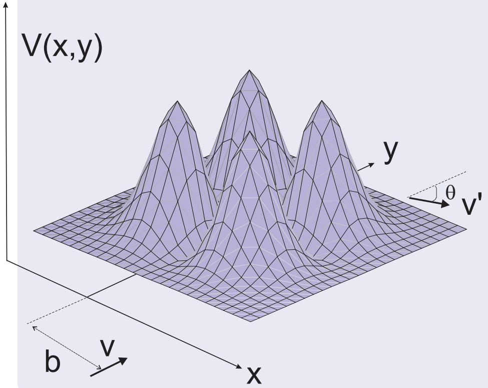

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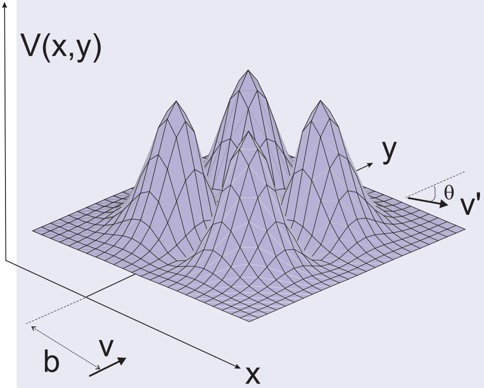

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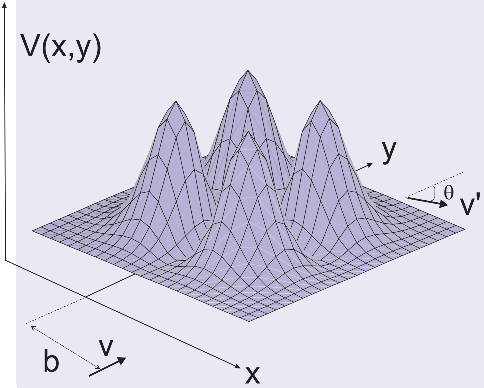

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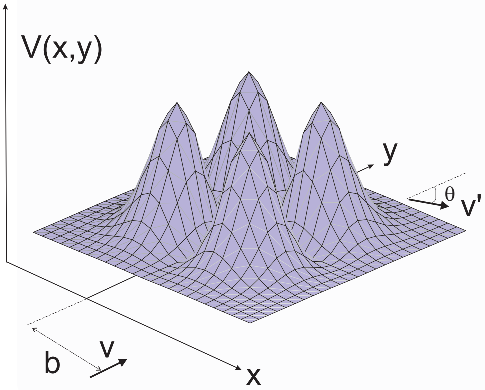

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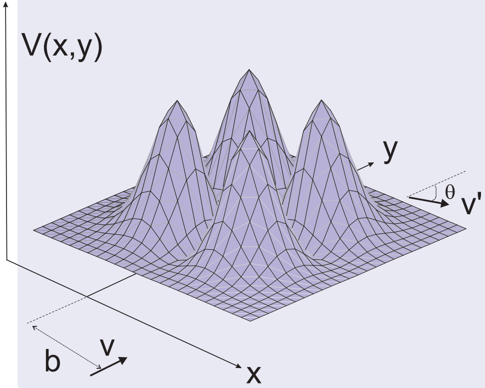

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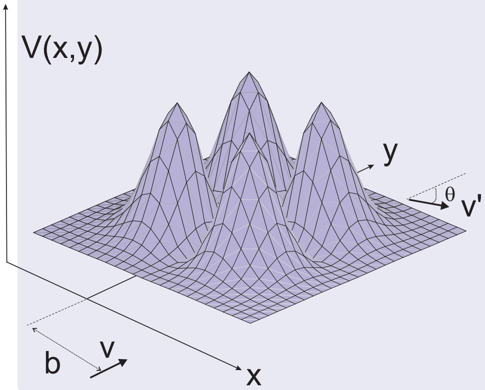

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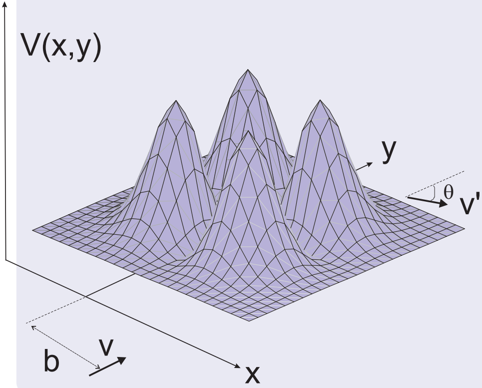

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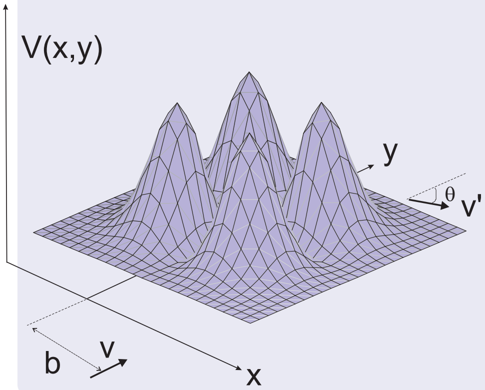

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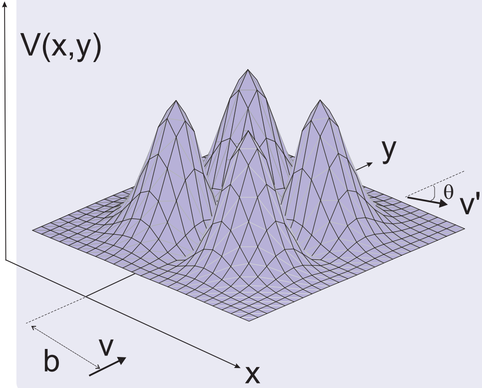

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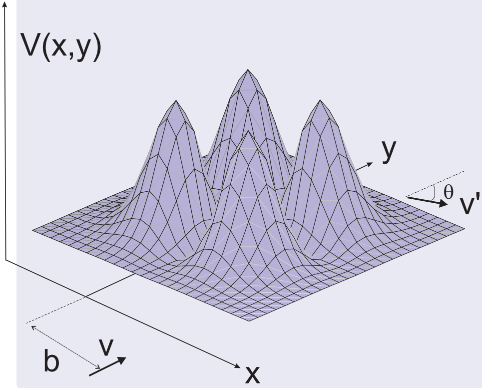

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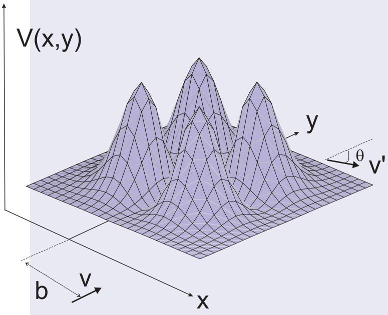

- Measure $N(\theta) \rightarrow \ \Rightarrow \sigma(\theta)$
- Differential cross section $\sigma(\theta)$
- Independent experiment details

$$\sigma(\theta) = \lim \frac{N_{\text{scatt}}(\theta)/\Delta\Omega}{N_{\text{in}}/\Delta A_{\text{in}}} \tag{1}$$

- Compare theory

$$\sigma(\theta) = \frac{b}{\left|\frac{d\theta}{db}\right| \sin\theta(b)} \tag{2}$$

- Unusual $d\theta(b)/db$

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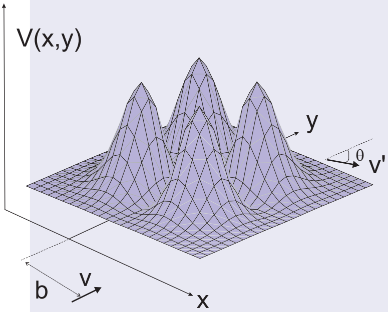

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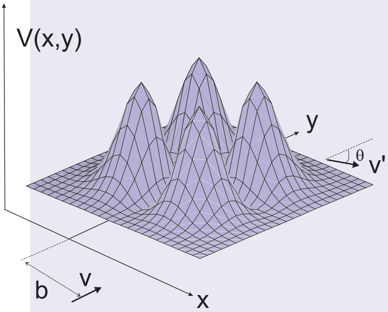

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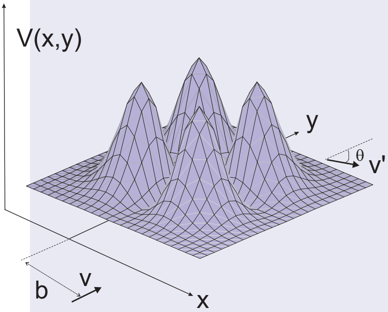

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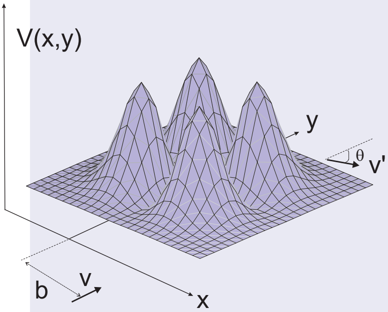

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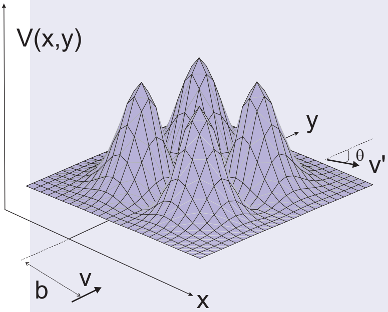

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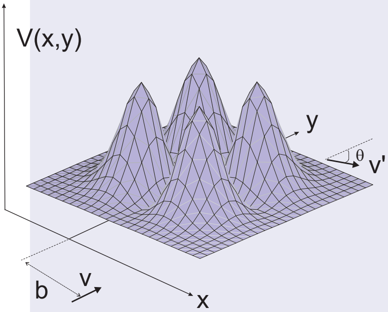

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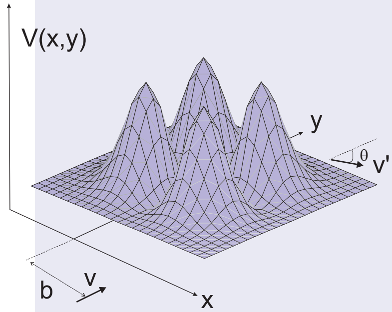

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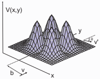
Theory: Equations to Solve

Newton's Law in x-y Plane

$$\mathbf{F} = m\mathbf{a}$$



$$-\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} = m\frac{d^2\mathbf{x}}{dt^2} \quad (1)$$

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- Simultaneous 2^{nd}-order ODEs

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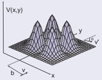

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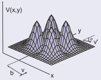

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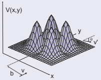

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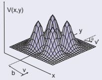

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Implementation Applet

Simultaneous $x(t)$, $y(t)$ rk4: Simultaneous 1^{st} $\mathcal{O}$

- $\Rightarrow$ Trajectory $[x(t), y(t)]$
- $\Rightarrow$ 4 1^{st} $\mathcal{O}$ ODEs:

$$\frac{d\mathbf{y}(t)}{dt} = \mathbf{f}(t, \mathbf{y}), \tag{1}$$

$$y^{(0)} \stackrel{\text{def}}{=} x(t), \qquad y^{(1)} \stackrel{\text{def}}{=} y(t), \tag{2}$$

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Force Function

$$f^{(0)} = y^{(2)}, \qquad f^{(1)} = y^{(3)}, \tag{4}$$

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Implementation Applet

- $\Rightarrow$ Trajectory $[x(t), y(t)]$
- $\Rightarrow$ 4 1^{st} $\mathcal{O}$ ODEs:

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Assessment

Apply rk4 With 4-D Force Function

1. **Initial Conditions:** $v_x = 0$, $x = b$ (impact param)
2. $m = 0.5$, $v_y(0) = 0.5$, $-1 \leq b \leq 1$
3. Plot trajectories $[x(t), y(t)]$
4. Look for $\sim$backward, multiple scattering
5. Phase space trajectories $[x(t), \dot{x}(t)]$, $[y(t), \dot{y}(t)]$
6. How scattering, bound problem differ?
7. Determine $\theta =$ `atan2(Vx,Vy)` ($y = +\infty$)
8. When discontinuous $d\theta/db$?
9. Run attractive & repulsive potentials
10. Run range E [$V_{max} = \exp(-2)$]
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