

The Chaotic Pendulum III

Double Pendulum Alternative

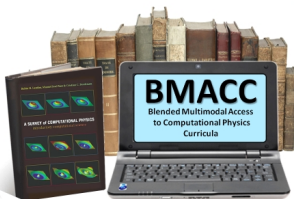
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Based on *A Survey of Computational Physics* by Landau, Páez, & Bordeianu

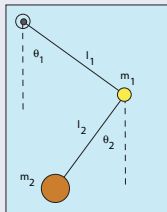
with Support from the National Science Foundation

Course: **Computational Physics II**



Double Pendulum: Alternative Problem

Chaos without External Torque or Friction



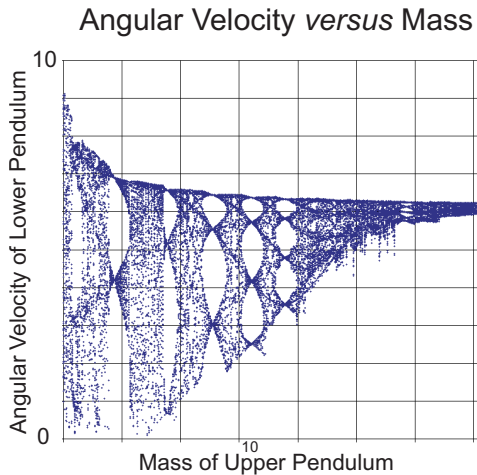
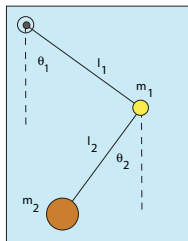
- No small- θ
- Coupling = extra degree freedom
- Small θ : in- ϕ , out- ϕ

$$L = \text{KE} - \text{PE} = (m_1 + m_2)l_1^2\dot{\theta}_1^2/2 + m_2l_2^2\dot{\theta}_2^2/2 \quad (1)$$
$$+ m_2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2) + (m_1 + m_2)gl_1 \cos \theta_1 + m_2gl_2 \cos \theta_2$$

$$\Rightarrow (m_1 + m_2)l_1\ddot{\theta}_1 + m_2l_2\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + m_2l_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) \quad (2)$$
$$+ g(m_1 + m_2) \sin \theta_1 = 0$$

$$m_2l_2\ddot{\theta}_2 + m_2l_1\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2l_1\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) + mg \sin \theta_2 = 0 \quad (3)$$

Double Pendulum: Bifurcations



Double Pendulum: Movies In-Phase, Out-Phase

loading mode1

loading mode2

Double Pendulum: Movie: Combined Large Oscillation

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