Extension of String Waves

Include Friction & Gravity

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Based on *A Survey of Computational Physics* by Landau, Páez, & Bordeianu

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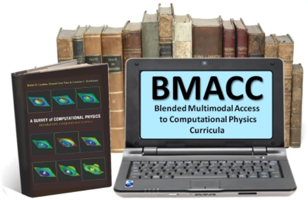
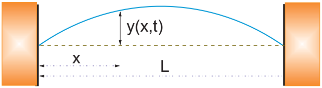
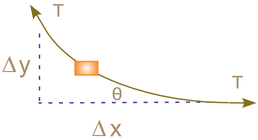
Course: **Computational Physics II**



Including Friction (Easy Numerically)





How Include Friction in Wave Equation?

- Observation: Real strings don't vibrate forever
- Model: String element in viscous fluid (κ)
- Frictional force opposes motion, $\propto v, \Delta x$:

$$F_f \simeq -2\kappa\, \Delta x\, \frac{\partial y}{\partial t} \tag{1}$$

- Modified wave equation (Additional RHS Force)

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} - \frac{2\kappa}{\rho} \frac{\partial y}{\partial t} \tag{2}$$

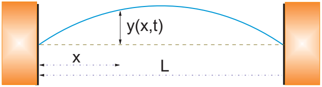
Exercise

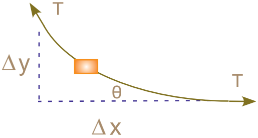
Do On Your Own

1. Generalize wave equation algorithm to include friction
2. Solve wave equation
3. Check that wave decays, or not ($\kappa = 0$)
4. Unstable: $\kappa < 0$?
5. Pick large enough κ to see effect; if too large?

loading CatFrictionAnimate

Another Extension: Variable Tension, Density





- $c = \sqrt{T/\rho}$;constant T, ρ
- $\Rightarrow$ fast & slow; adiabatic
- g, $\rho(x)$ $\Rightarrow$ $T(x)$, $c(x)$
- $\uparrow \rho \Rightarrow \uparrow T$ to accelerate
- Thick chain ends; g
- Newton for element:

$$F = ma \tag{3}$$

$$\frac{\partial}{\partial x}\left[T(x)\,\frac{\partial y(x,t)}{\partial x}\right]\Delta x = \rho(x)\Delta x\,\frac{\partial^2 y(x,t)}{\partial t^2} \tag{4}$$

$$\frac{\partial T(x)}{\partial x}\,\frac{\partial y(x,t)}{\partial x} + T(x)\,\frac{\partial^2 y(x,t)}{\partial x^2} = \rho(x)\,\frac{\partial^2 y(x,t)}{\partial t^2} \tag{5}$$

Discretized Wave Equation with $T(x)$

Trial Case: $\rho(x) = \rho_0 e^{\alpha x}, \quad T = T_O e^{\alpha x}$

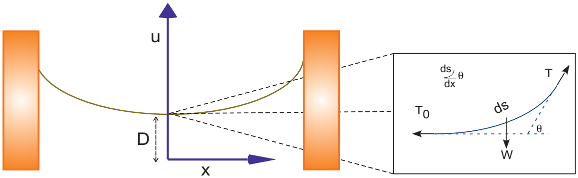
$$\rho(x) \frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial T(x)}{\partial x} \frac{\partial y(x,t)}{\partial x} + T(x) \frac{\partial^2 y(x,t)}{\partial x^2} \quad (1)$$

- Difference equation via central-difference derivatives:

$$y_{i,2} = y_{i,1} + \frac{c^2}{c'^2}[y_{i+1,1} + y_{i-1,1} - 2y_{i,1}] + \frac{\alpha c^2 (\Delta t)^2}{2\Delta x}[y_{i+1,1} - y_{i,1}] \quad (2)$$

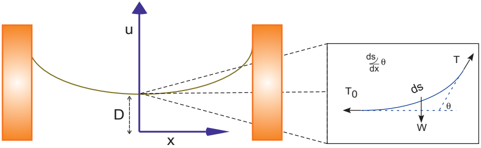
- Try standing waves $y(x,t) = A\cos(\omega t)\sin(kx)$
- Verify $\omega \leq \omega_{cut} \Rightarrow$ no solution

Hanging String at Rest: Derivation of Catenary



- Chains sag
- $T(x)$: ends support middle
- $u(x)$ = equilibrium shape,
  $y(x)$ = disturbance
- $u(x)$, $T(x)$ =?
- Free-body diagram
- W =section weight = T_y
- s = arc length:

Hanging String: Derivation of Catenary (Cont)



$$T(x)\sin\theta = W = \rho g s, \quad T(x)\cos\theta = T_0, \tag{1}$$

$$\Rightarrow \quad \tan\theta = \rho g s / T_0 \tag{2}$$

Trick: convert to ODE, solve ODE (see text)

$$u(x) = D\cosh\frac{x}{D} \qquad \text{NB Special Origin)} \tag{3}$$

$$T(x) = T_0\cosh\frac{x}{D}, \quad D \stackrel{\text{def}}{=} T_0/\rho g \tag{4}$$

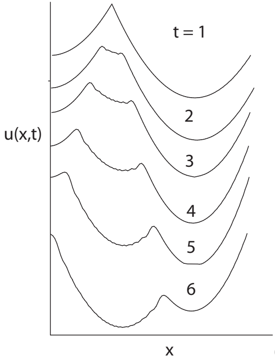
Now know $T(x)$ for wave equation

$$\frac{\partial T(x)}{\partial x}\frac{\partial y(x,t)}{\partial x} + T(x)\frac{\partial^2 y(x,t)}{\partial x^2} = \rho(x)\frac{\partial^2 y(x,t)}{\partial t^2} \tag{5}$$

Implementation: Catenary and Frictional Waves

1. Modify EqString.py to include friction and g
2. Look for interesting cases, create surface plots
3. Point out non uniform damping.
4. Are standing waves (normal modes) possible?

$$u(x,t) = A\cos(\omega t)\sin(\gamma x)$$



(Get to work!)